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Sudden Stops and Collateral Constraints: Searching for "Wally"

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Sudden Stops and Collateral Constraints: Searching for "Wally" [1]
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Abstract: Is there a connection between a sudden stop and debt issuance per year? We provide a positive answer to this question using a novel database. We find that countries that experienced a current account deficit of 7% or more during 2 to 3 year will suffer a sudden stop measured by a 4.7% to 5.0% consumption drop and a 4.0% current account reversal. Similarly, countries that experienced a current account deficit of 6% of the GDP or more during 4 to 5 years will suffer a sudden consumption drop ranging between 4.4% and 4.9% and a current account reversal between 3.2 and 3.8%. These findings serve can be used as a leading indicator for this type of events. Moreover, using a novel recursive equilibrium notion due to Pierri and Reffet (2018) we are able to match the event using a simple model without imposing shocks to deep parameters or an additional structure to exogenous variables, as it is sometimes done in the literature. The method captures completely the multiplicity of equilibria latent in the sequential equilibrium and provides evidence in favor of interpreting a sudden stop as a coordination event, similar to a bank run.

JEL CODE: F32, F41

I. Introduction

In May 2018 Argentina suffered yet another currency crisis, generating a severe adjustment in private expenditure. There is consensus that the cause of this event was an unexpected reversal of capital flows that followed after a debt issuance of around 15% of the GDP in a time span of less than 1.5 years. The question that arises, then, si the following: *is there a connection between sudden stops and debt issuance per unit of time?* This paper provides an affirmative answer to this question. We find that a current account deficit of or above 7% of the GDP per year can be sustained for at most 3 years. Similarly, a deficit of at least 6% can last no more than 5 years. Despite the heterogeneity that characterize these thresholds,

[1] The title refers to the puzzle books *Where's Wally?* (*Waldo* in the US version) Filled with illustrations depicting many people at a given location, readers are challenged to find a character named Wally hidden in the population.
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the disruption which follows them has a similar structure and can be measured, roughly, by a 5% drop in consumption and a 4% current account reversal.

The contrast between the heterogeneity of the events that precedes the sudden stop and the similarity in the magnitude of the crises as measured by changes in domestic private expenditure and current account suggests the possibility of a coordination episode in a context of multiple equilibria, after controlling for possible heterogeneity in the endowments or parameters which characterize the economy. We interpret this observation as follows: the accumulation of current account deficits in the presence of market frictions gives rise to a continuum of possible continuation paths every time that the county faces a significant debt issuance. Market participants can coordinate in a sudden stop of different magnitudes if frictions become relevant. This last fact can be captured by a binding price-dependent liquidity constraint, sometimes referred as “collateral requirements”. The resemblance in the observed consumption drop suggests that “the market” considers sufficient a 5% drop in consumption to correct the disequilibrium. We show that a novel recursive equilibrium notion due to Pierri and Reffet (2018) is able to capture the latent multiplicity of the sequential equilibrium in a canonical model due to Bianchi (2011). This result allows us to match the observed behavior, a fact that has been elusive to the literature even allowing for a structural break in the deep parameters (see Seoane, 2018).

From a policy perspective, the empirical results in this paper can serve as a leading indicator of sudden stops. In the presence of persistent and significant external debt issuance, local authorities can take macro-prudential measures if, for example, the current account deficit is around some of the thresholds found in this paper. From a methodological perspective, as we find evidence in favor of a more restrictive definition of a sudden stop and propose a more flexible recursive equilibrium notion, compared to the ones that are currently being used in the literature, we can improve the match of available models without requiring additional structure in the exogenous variables nor the parameters. From a theoretical perspective, this paper provides evidence in favor of interpreting a sudden stop as a coordination event in the presence of incomplete capital markets and multiple equilibria.

The recent events in Argentina and Turkey are not the only ones in modern history. These episodes have been frequent in emerging markets for the last 40 years. In principle, it seems that they differed not only in the prelude of the sudden stop but also in the size of the crises that followed it. The canonical event was the 1995 Mexican crisis: after the capital flow reversal, the current account shifted by almost 10% of GDP, and private expenditure as well as output fell by almost the same magnitude as in the Great Depression. Before that, the country experienced a current account deficit of more than 5% of the GDP during 4 years. That is, an increase of external debt of almost 20% of the GDP in that time span. On the contrary, Argentina issued almost 15% of the GDP in 1.5 years, a figure that looks decoupled from the accumulated current account deficit (8% of the GDP) due to the “hold-out problem” (see Buchheit *et al.* 2013), and suffered a sudden stop of magnitudes that are yet to be discovered. This paper shows that even though the “pre-phase” of a sudden stop differ greatly across countries, the collapse which follows the event has fewer variance.

We interpret this difference in the variability between the prelude of a sudden stop and the disruption which follows it as a coordination outcome out of possible multiple equilibria. This explanation, in line with the other pieces of the literature such as Schmitt-Grohé and Uribe (2017), is not only theoretically enriching but also methodologically useful as it allows to us improve the match of standard models; a fact that has been slippery to the applied literature (see Seaone and Yurdagul, 2018). In particular, using a definition that match the criteria proposed by Edwards (2004), Calvo (2008) and Mendoza (2010) we restrict the number of episodes that qualify as a sudden stop. Moreover, the mentioned recursive equilibrium notion proposed by Pierri and Reffett (2018) endows the model with sufficiently flexibility to match the observed behavior without requiring shocks to deep parameters nor additional assumptions about the exogenous variables. This is possible due to the capacity of the recursive equilibrium notion to capture in full extent the latent multiplicity of equilibria in the sequential economy.

Finally, the empirical characterization of the "pre-phase" of a sudden stop could serve as a leading indicator of the event. Taking into account the increasing interest in the literature in the macro-prudential measures (see for instance, Bianchi and Mendoza, 2018) the results in this paper could serve to detect future collapses in foreign borrowing as we describe the time path that typically precedes sudden stops as a function of the time rate of external debt issuance.

The paper is organized as follows. Section II presents several definitions of a sudden stop and places the paper in the literature. Section III presents the formal theoretical model and, the recursive and sequential equilibrium definition and illustrate how they can be used to match a typical sudden stop. Section IV describes the empirical procedure used to find sudden stops and characterize the "pre and post" phases. Section V concludes and anticipate some clues to our future work in this subject.

II. Sudden Stops, Debt and the Collateral Constrain

Our quest starts empirically, using an ample database for several countries to identify a broad definition of Sudden Stops (SS); one that encompasses the spirit of Calvo (1998), Edwards (2004) and Mendoza (2010). In particular, we look for episodes that comprises at the same time the following properties: (i) a sharp drop in (total) real consumption; (ii) several years of accumulated current account deficits; and (iii) an acute correction of the current account following the SS episode. This is the "Wally" we are eager to find.

Although the concept of a SS was originally defined by Calvo (1998), there have been several "practical" definitions, and the disparity among them can be seen not only in the magnitude of the collapse which follows the SS but also in whether they consider or not the "pre-phase" as a mandatory requirement. Among the most relevant ones, we found:

i) Mendoza (AER, 2010):

Three main empirical regularities define Sudden Stops: (i) reversals of international capital flows, reflected in sudden increases in net exports and the current account, (ii) declines in production and absorption, and (iii) corrections in asset prices.

ii) Mendoza and Smith (JIE, 2006)

*A Sudden Stop is defined by three stylized facts: sudden, sharp reversals in capital inflows and the current account, large declines in absorption and production, and collapses in real asset prices and in the price of non-tradable goods relative to tradables (*see table in page 83 for an empirical representation of this definition)

iii) Calvo et al. (NBER, 2008)

These are episodes in which the economy exhibits a "large and largely unexpected" cut in capital inflows. In addition, we zero in on "systemic" Sudden Stops (3S), i.e., sudden stops that take place in conjunction with a sharp rise in aggregate interest-rate spreads.

iv) Calvo *et al.* (IADB WP, 2004)

We first define a Sudden Stop as a phase that meets the following conditions:

• *It contains at least one observation where the year-on-year fall in capital flows lies at least two standard deviations below its sample mean (this addresses the "unexpected" requirement of a Sudden Stop).*

• *The Sudden Stop phase ends once the annual change in capital flows exceeds one standard deviation below its sample mean. This will generally introduce persistence, a common fact of Sudden Stops.*

• *Moreover, for the sake of symmetry, the start of a Sudden Stop phase is determined by the first time the annual change in capital flows falls one standard deviation below the mean*

v) Edwards (NBER, 2004)

I defined a "sudden stop" episode as an abrupt and major reduction in capital inflows to a country that up to that time had been receiving large volumes of foreign capital. More specifically, I imposed the following requirements for an episode to qualify as a "sudden stop": (1) The country in question must have received an inflow of capital larger to its region's third quartile during the previous two years prior to the "sudden stop." And (2), net capital inflows must have declined by at least 5% of GDP in one year.

Notice that, except for definition v) due to Edwards, the remaining concepts focus on the SS and the "post-phase". We take the route chosen by Edwards, the most restrictive one, because that will allow us to match the empirical performance of the selected model by reducing the number of possible events. Moreover, we do not impose an ex-ante criteria neither for the pre-phase nor for the SS. We allow for multiple thresholds as measured by the combination of a minimum level of current account deficit and the consecutive years which the actual figure were at or above it. The countries matching this criteria are placed in the "treatment group". We then draw a curve by mapping the difference between the change in the current account in the treatment and the control group against the minimum current account deficit in the threshold. The SS is then identified by the "biggest" reversion in capital flows, measured by the steepest tangent of this curve.

In this paper we apply a novel recursive equilibrium notion due to Feng et al. (2015), and to a slightly modified version of the model in Schmitt-Grohé and Uribe (SGU, 2017). The model describes a small open economy subject to liquidity constraints and incomplete markets. In this framework, a SS is modeled as an unanticipated change in the parameters. Specifically, liquidity restrictions are represented as a price dependent inequality constraint that affects external debt issuance in the form of a maximum "debt to GDP ratio". Once this inequality is binding, it is possible to generate the desired current account reversal due to a "shift" in the maximum permissible level of debt. This procedure have been shown to be insufficient to fully capture the anatomy of a SS in a standard minimal state space recursive representation as in Bianchi (2011) (see for instance Seone and Yordagul, 2018). This paper shows, using the results in Pierri and Reffet (2018), that it is possible to apply the Feng et al. recursive equilibrium notion to this framework in order to improve the empirical performance of the model. Thus, we are placing this paper in the middle between the structural macro and the theoretical literature.

Typically, a SS is seen as a rapid, unexpected, and coordinated reversal of international capital flows[3]. But this definition needs to clarify which exactly is the signal that help lenders to converge in their expectations. One possibility is to rely in a “sunspot”, as defined by Reinhart and Rogoff (2008). The recursive equilibrium notion used in this paper allows us to use this interpretation to match the observed SS. Contrarily to what have been done in other pieces of the theoretical literature (see for instance Duffie et al., 1994), the sunspot in this case does not serve a technical purpose (i.e. convexify a correspondence). Pierri and Reffett (2018) shows that applying Feng *et al*.’s results to the Bianchi (2011) framework gives rise to a continuum of possible equilibiria *in the presence of price dependent inequality constraints*. Thus, sunspots arise naturally as a coordination mechanism which, in turn, improves significantly the “fit” of the model with data without requiring, for instance, a shift in the permissible debt to GDP ratio.

III. A Structural Model

III.1 Definitions and characterization

The model is a sequential version of Bianchi (2011). Contrarily to what is done in SGU (2017), we choose a primal characterization of an otherwise standard sequential equilibrium definition. The reason behind this type of representation is that it gives rise to a flexible markovian equilibrium notion, borrowed from Feng *et. al.* (2015), which in turn allows to match not only a sudden stop of different magnitudes but also the dynamic behavior that typically precedes this type of events; both under a standard parametrization.

The “backwards” notion of Feng *et. al.*’s recursive equilibrium, inherited from Duffie *et. al.* (1994), allows us to accommodate easily the distinctive feature of a SS, namely, a real exchange rate appreciation and debt accumulation followed by a drop in absorption and a current account reversal. This can be done by assuming, in the presence of a collateral constraint, that the sudden stop is characterized by 2 consecutive hits in the borrowing constraint. This fact is not easily accommodated by a dual representation of the sequential equilibria as the one traditionally used in the literature (see Bianchi 2011, Mendoza 2010, Mendoza and Smith, 2006) as it is difficult to identify the effects of endogenous variables, such as prices, on the lagrange multipliers.

We start by assuming a small open endowment economy populated by an infinitely lived representative agent who consumes tradable and non-tradable goods, respectively c^T, c^N. The relative price of non-tradables is denoted by p. The agent can also borrow from abroad an internationally traded liability d (if $d < 0$ we say that the agent holds a net foreign asset), paying a constant interest rate r. This type of trade is restricted by a borrowing constraint where the ratio of debt to current income, measured in tradable units, must lie bellow $\kappa > 0$. The borrowing constraint is interpreted to be a collateral constraint and the pledgeable object is a fraction κ of income. Let $y^T + py^N$ be the total income in any given period, then the principal paid by the agent is given by $min\{d, \kappa(y^T + py^N)\}$. We assume that y^N is constant and y^T follows a finite state, Y, Markov process with transition matrix q. Finally, preferences

[3] In dollarized economies, a non neglibible component of the outflows during the crisis is capital flight by residents, in the form of dollar savings out of the legal financial system.

are the standard in the literature: the *intertemporal* problem is characterized by a CRRA instantaneous return function with parameter $\sigma > 0$, while the *intratemporal* problem assumes CES preferences with parameter $\gamma > 0$. As will be clear from the FOCs of the model, we require $\gamma + \sigma > 1$ in order to guarantee a well behaved Euler equation. Taking into account the parametrization used in the literature (see Bianchi, 2011), this restriction is mild.

The agent solves the following maximization problem:

Problem 1

$$Max\ E_0 \sum_{t=0}^{\infty} \beta^t [C_t^{1-\sigma} - 1] / (1 - \sigma)$$

Subject to

$$C_t = [(c_t^T)^\gamma + (c_t^N)^\gamma]^{1/\gamma}$$

$$c_t^T + P_t c_t^N + (1+r) d_t = y_t^T + P_t y_t^N + d_{t+1}$$

$$d_{t+1} \geq \kappa [y_t^T + P_t y_t^N]$$

$$d_0 \in \mathbb{R}, y_0^T \in Y, y_t^N = y^N > 0$$

Where the second equation is the flow budget constraint and the third is the collateral requirement.

Now we are in position to define a (sequential) competitive equilibrium. Recall that randomness in this economy comes from the markovian process that drives the tradable endowment. This guarantees the existence of an infinite horizon process for $\{y_t^T\}$, $(\Omega, \mathcal{F}, \mu_{y_0^T})$, where Ω is the sample space which lies in the space of infinite bounded sequences, $\mathcal{F}$ is the associated sigma algebra with filtration $\mathcal{F}_t$ and $\mu_{y_0^T}$ is the associated probability measure for a given initial condition, y_0^T (see for instance Stokey, Lucas and Prescott, 1989, Ch. 7).

Definition 1: Sequential Competitive equilibrium (SCE)

A SCE for this economy is composed by 4 progressively $\mathcal{F}_t$-measurable functions $p, (c^T, c^N, d)$ such that:

i) Given $p,\ (c^T, c^N, d)$ solves problem 1
ii) For each $t, \omega_t \in \mathcal{F}_t$; $c^T(\omega_t) = y^N$

Pierri and Reffett (2018) provide sufficient conditions to guarantee the compactness of the SCE, a fact that is critical to approximate the model numerically. The authors showed that it suffice to impose bounded marginal utilities and $\beta(1+r) < 1$. As the preference structure in problem 1 do not match these assumptions, we must impose the existence of an upper bound on debt, $D > 0$, which is never binding. Under this assumption, we can assure that for each $t, \omega_t \in \mathcal{F}_t$, $[p, (c^T, c^N, d)](\omega_t) \in K$ with $K \subset \mathbb{R}^4$ compact.

Now we are allowed to characterize a (compact) SCE by means of a sequence of FOCs. As mentioned before, we choose a primal version of the Karush-Kuhn-Tucker conditions because they allow to understand the dynamic behavior of the model using a flexible markovian representation that follow directly from them.

We look for a system of equations that characterize definition 1. Thus, for each $t, \omega_t \in \mathcal{F}_t$, $[y^T, p, d](\omega_t) = [y_t^T(\omega_t), p_t(\omega_t), d_{t+1}(\omega_t)]$ must satisfy:

$$\left[\left[X_t(p_t, d_{t+1}) - \beta(1+r)E_t\big(X_t(d_{t+1})\big) \right] \left[d_{t+1} - \kappa(y_t^T + p_t y^N) \right] \right](\omega_t) = 0 \qquad (1)$$

$$p_t = \left[\frac{y_t^T + min\{d_{t+1}, \kappa(y_t^T + p_t y^N)\} - (1+r)d_t}{y^N} \right]^{1-\gamma} (\omega_t) \qquad (2)$$

The preference structure implies that $X_t(\omega_t)$, for each $t, \omega_t \in \mathcal{F}_t$, must satisfy:

$$X_t(\omega_t) = [(c_t^T(\omega_t))^\gamma + (y^N)^\gamma]^{\frac{1-(\gamma+\sigma)}{\gamma}} + [(c_t^T(\omega_t))^\gamma]^{\gamma-1} \qquad (3)$$

$$c_t^T(\omega_t) = [y_t^T + min\{d_{t+1}, \kappa(y_t^T + p_t y^N)\} - (1+r)d_t](\omega_t) \qquad (4)$$

Once the compactness of the SCE is established, we can borrow the recursive structure in Pierri and Reffett (2018). In particular, we arrive to the following definition of recursive equilibrium.

Definition 2: Recursive Competitive equilibrium (RCE)

Let $z_t \equiv [y_t^T, p_t, d_t]$ *and* Φ *a correspondence mapping* $K \mapsto K$. *We say that* $\Phi(z_t) \in z_{t+1}$ *if for each* $y_{t+1}^T \in Y$ *there exist* $z_{t+1}(y_{t+1}^T) \in K$ *and* $E_t\big(X_t(z_{t+1})\big)$ *such that* z_t *satisfies equations (1)-(4).*

The existence of this type of equilibria is proved in Pierri and Reffet (2018) (see Lemma 1).

First, notice the backwards notion of Definition 2. Contrary to the standard definition of RCE in, for instance, Mehra and Prescott (1980), our strategy consists of "picking up" a continuation and see if there exist a suitable predecessor. The other critical departure of our model from the standard recursive literature is that the state space is different but not necessarily smaller. Bianchi (2011) deals with $[y_t^T, D_t, d_t]$, where D_t is the "aggregate state", typically associated with overall market restrictions. But as discussed in Pierri and Reffet (2018), this type of recursive equilibria is more restrictive than the one presented in Definition 2. From a theoretical point of view, these restrictions generate the lack of ergodicity of the Markov equilibria and from a practical point of view they cause a poor match between model and data (see Seoane *et al.*, 2018).

III.2 The anatomy of a sudden stop

In this section we want to illustrate that Definition 2 is flexible enough to match the anatomy of a sudden stop, as has been defined in section II and will be described in section IV.

Specifically, our method entails matching a decrease in c_{t+1}^T, in p_{t+1} and in $(d_{t+2} - d_{t+1})$ that is preceded by an increasing sequence of $\{p_{t-i}, d_{t-i}\}_{i=1}^{\tau}$, and a weakly increasing sequence of tradable consumption $\{c_{t-i}^T\}_{i=1}^{\tau}$, where $\tau = 2,..,6$ and $c_{t-i}^T \geq \underline{c} > 0$. The last assumption, although it serves a technical purpose, captures the idea that the country is borrowing abroad to sustain a certain level of tradable consumption typically associated with basic necessities.

The non-binding versions of equations (1) and (2) can be written as follows:

$$X_t(d_{t+1}) - \beta(1+r)E_t\big(X_t(d_{t+1})\big) = 0$$

$$p_t = \left[\frac{y_t^T + d_{t+1} - (1+r)d_t}{y^N}\right]^{1-\gamma}$$

Note the "sequential" form of this system. In equilibrium, d_{t+1} is determined independently of p_t and p_{t+1} which means that the intertemporal behavior, given by the Euler equation, can be computed from equation 1, and then a suitable price can be recovered from equation 2. Further, as d_{t+1} is independent of y_{t+1}^T and $\beta(1+r)E_t\big(X_t(d_{t+1})\big)$ does not depend on $p_{t+1}(y_{t+1}^T)$, the backwards nature of Definition 2 does not really has a distinctive effect on the structure of equilibria.

Formally, using Definition 1, we see that d_{t+1} must be $\mathcal{F}_t$-measurable or equivalently, it must be constant with respect to y_{t+1}^T. Thus Definition 2 implies that we have to choose d_{t+1} and $p_{t+1}(y_{t+1}^T)$ for y_{t+1}^T in Y such that it satisfies the "$t+1$" version of equation (2). Note that d_{t+2} is not restricted at all by the optimal choice in period "t" and the only restriction is given by the intratemporal optimality condition in equilibrium:

$$d_{t+2} \in \left\{d \in K \,|\, \frac{y_{t+1}^T + d - (1+r)d_{t+1}}{y^N} > 0\right\} \tag{5}$$

When the collateral constraint is non-binding, the SCE is a standard savings problem and all suitable recursive equilibrium notions must reflect that fact. In this scenario, any positive y_t^T-shock will be smoothed, implying an improvement in the current account along with the required exchange rate appreciation to satisfy equation (2), which indicates that p is increasing in c_t^T. A negative y_t^T-shock has the symmetric effect, making the non-binding version not suitable to describe the "pre-phase" of a sudden stop.

In order to match the decrease in c_{t+1}^T observed in a sudden stop, the real exchange rate must depreciate in the same period, due to the monotonic nature of equation (2), so we can choose $\tilde{d}_{t+1} > d_{t+1}$. However, d_{t+2} is not really "pin down" by Definition 2, as can be seen from equation (5). Thus, the non-binding version of equations (1)-(4) do not seem suitable to match a concrete order of magnitude in the observed variables (for instance, a 5% drop in tradable consumption).

Fortunately, the presence of a binding collateral constraint allows news possibilities. When collateral is present, equations (1) and (2) become:

$$X_t(p_t) \geq E_t[X_{t+1}(p_t)](1+r)\beta$$

$$p_t = \left[\frac{y_t^T + \kappa(y_t^T + p_t y^N) - (1+r)d_t}{y^N}\right]^{1-\gamma}$$

In this new setup, Definition 2 implies that given a sequence $\{z_{t+1}(y_{t+1}^T)\}_{y_{t+1}^T \in Y}$ in K, we are allowed to use equation (1) to compute p_t, and then find a suitable d_t from equation (2), provided that we are sufficiently far away from the boundary of K. The shape of the system looks promising as we are breaking the "sequential" nature of the non-binding scheme. Note that now d_{t+1} is tied up with p_t by the collateral constraint, and this allow us to connect more tightly the intertemporal behavior of the model with the sequence of equilibrium prices.

Now we want to go one step further and resemble the anatomy of a sudden stop. One common feature of sudden stops is the reversal of the current account caused by a drop in tradable consumption. We want to show that $(c_{t+1}^T - c_t^T)/c_t^T \approx -a$ and $(d_{t+2} - d_{t+1})/d_{t+1} \approx -b$, where a and b are 2 positive real numbers. The standard procedure in the literature is that the collateral binds only for on period, d_{t+1}. However, what we typically observe in a sudden stop is a significant drop in consumption happening in period $t+1$ when the current account reach certain threshold in period t, so the event involves two consecutive budget constraint periods. Thus, both d_{t+1} *and* d_{t+2} becomes relevant to describe the anatomy of the sudden stop.

Fortunately, this can be done without loss of generality as Pierri and Reffet (2018) showed that the system hits the collateral constraint with positive probability for any initial condition (y_0^T, d_0).

When both d_{t+1} *and* d_{t+2} becomes relevant and that the collateral constraint hits the economy for 2 consecutive periods, we have $d_{t+1} = \kappa(y_t^T + p_t y^N)$, $d_{t+2} = \kappa(y_{t+1}^T + p_{t+1} y^N)$ and $d_t > 0$. Plugging these assumptions into equations (1) and (2) we obtain:

$$[(y_t^T + \kappa(y_t^T + \boldsymbol{p_t} y^N) - (1+r)d_t)^\gamma + (y^N)^\gamma]^{\frac{1-(\gamma+\sigma)}{\gamma}} + [(y_t^T + \kappa(y_t^T + \boldsymbol{p_t} y^N) - (1+r)d_t)^\gamma]^{\gamma-1}$$

$$> \qquad (1')$$

$$\sum_{y_{t+1}^T \in Y} q(y_t^T, y_{t+1}^T)[\left(y_{t+1}^T + \kappa(y_{t+1}^T + \boldsymbol{p_{t+1}} y^N) - (1+r)\kappa(y_t^T + \boldsymbol{p_t} y^N)\right)^\gamma + (y^N)^\gamma]^{\frac{1-(\gamma+\sigma)}{\gamma}}$$

$$+ [\left(y_{t+1}^T + \kappa(y_{t+1}^T + \boldsymbol{p_{t+1}} y^N) - (1+r)\kappa(y_t^T + p_t y^N)\right)^\gamma]^{\gamma-1}$$

$$\boldsymbol{p_{t+1}} = \left[\frac{y_{t+1}^T + \kappa(y_{t+1}^T + \boldsymbol{p_{t+1}} y^N) - (1+r)\kappa(y_t^T + \boldsymbol{p_t} y^N)}{y^N}\right]^{1-\gamma} \qquad (2')$$

Since now we are assuming the existence of an upper bound that is never binding, we are free to choose any continuation $z_{t+1} \equiv [y_{t+1}^T, p_{t+1}, d_{t+1}] \in K$. Take for example $\tilde{P}_{t+1}(y_{t+1}^T) < P_{t+1}(y_{t+1}^T)$ for all y_{t+1}^T in Y. As we explain below, this implies that a $\tilde{P}_t > P_t$ exists for equation (1'). To see this, note that the Euler equation holds with strict inequality as we are assuming that the collateral constraint binds in period "t". The assumption $d_t > 0$, guarantees that there always exists a $\tilde{d}_t < d_t$ which satisfies the LHS of the same equation together with the "time t" version of equation (2'). Given a uniform bound on tradable consumption, we have $dp_t/d(d_t) < 0$[4] . In order to complete the argument, we need to show that the pair $(\tilde{P}_{t+1}, \tilde{P}_t)$ satisfies the time $t+1$ version of equation (2'). This can be shown straightforwardly by

[4] Given $y^N = 1$, the condition holds provided that $c_t^T(\omega_t) > [(1-\gamma)\kappa]^{1/\gamma} \geq \underline{c}$. Pierri and Reffett (2018) provide sufficient conditions for the existence of a uniform lower bound on tradable consumption.

applying the implicit function theorem to equation (2'). Provided that tradable consumption is uniformly bounded away from zero[5], $dp_{t+1}/dp_t < 0$.

If we are enough far away from the boundary of K (something that can be verified numerically), we can push $\tilde{P}_{t+1}$ sufficiently away from P_{t+1} such that $\tilde{c}^T_{t+1} < c^T_{t+1}$, $\tilde{c}^T_t > c^T_t$, $\tilde{d}_{t+2} < d_{t+2}$ and $\tilde{d}_{t+1} > d_{t+1}$. Empirically, this conditions meet the anatomy of sudden stops, because they imply a drop in consumption and a current account reversal. Note that $\tilde{P}_{t+1} < \tilde{P}_t$ (i.e., a depreciation of the real exchange rate), is also present in several definitions of a sudden stop.

So this setup allows us to capture the anatomy of a sudden stop *without assuming shocks to the fraction of pledgeable debt, κ, nor to the current tradable income, y^T_t*, as it is frequently done in the literature (see Seoane *et al.* 2018 for a detailed discussion on this procedure). Moreover, this result highlights the importance of dealing with multiple equilibria in macroeconomics as the sequential version of one of the simplest models in the literature is able to capture the observed behavior, provided that we are willing to allow for more flexible markovian equilibrium notions Thus, as it is suggested by Calvo *et. al.* (2008) or Edwards (2004) a sudden stop can be thought as a coordination phenomenon, similar to a bank run.

What remains to be done is to match the "pre-phase" of a sudden stop, the manifestation of persistent current account deficits, real exchange rate appreciation and a (possibly weakly) growing economy before the event. Unfortunately, as discussed before, the non-binding version of equations (1)-(4) will not allow us to match the observed behavior. Thus, we need to look at the "binding" version of Definition 1. Assume that the collateral constraint binds in only period "$t+1$". Then, the system of equations (1)-(2) becomes:

$$[(y^T_t + \boldsymbol{d_{t+1}} - (1+r)d_t)^\gamma + (y^N)^\gamma]^{\frac{1-(\gamma+\sigma)}{\gamma}} + [(y^T_t + \boldsymbol{d_{t+1}} - (1+r)d_t)^\gamma]^{\gamma-1}$$

$$= \qquad (1'')$$

$$\sum_{y^T_{t+1}\in Y} q(y^T_t, y^T_{t+1})[(y^T_{t+1} + \kappa(y^T_{t+1} + \boldsymbol{p_{t+1}} y^N) - (1+r)\boldsymbol{d_{t+1}})^\gamma + (y^N)^\gamma]^{\frac{1-(\gamma+\sigma)}{\gamma}}$$
$$+ [(y^T_{t+1} + \kappa(y^T_{t+1} + \boldsymbol{p_{t+1}} y^N) - (1+r)\boldsymbol{d_{t+1}})^\gamma]^{\gamma-1}$$

$$\boldsymbol{p_{t+1}} = \left[\frac{y^T_{t+1} + \kappa(y^T_{t+1} + \boldsymbol{p_{t+1}} y^N) - (1+r)\boldsymbol{d_{t+1}}}{y^N}\right]^{1-\gamma} \qquad (2'')$$

Again we can choose freely $\{z_{t+1}(y^T_{t+1})\}_{y^T_{t+1}\in Y}$, we can pick $\tilde{p}_{t+1}(y^T_{t+1}) > p_{t+1}(y^T_{t+1})$ for $y^T_{t+1} \in Y$ which in turn implies, as the Euler equation holds with equality in this case, $\tilde{d}_{t+1} > d_{t+1}$. Then, the budget constrain equation in equilibrium in period "t" ($\tilde{c}^T_t + (1+r)d_t = y^T_t + \tilde{d}_{t+1}$) implies the desired increase in tradable consumption coupled with a current account deficit *for the same level of current tradable income*. Moreover, the increase in $\tilde{c}^T_t$ that follows from equation (2) implies the observed real exchange rate appreciation. Finally, depending

[5] Given $y^N = 1$, the condition holds provided that $c^T_t(\omega_t) > [(1-\gamma)/\kappa]^{1/\gamma} \geq \underline{c}$.

on $\tilde{p}_{t+1} - p_{t+1}$, equation (2) implies that we can always find $\tilde{d}_t > d_t$ and we can generate the desired persistence in the current account deficit for periods $t-1, \ldots, t-\tau$.

One final remark concerns the behavior of exchange rate. Note that equation (2) implies that, as we are defining a SS to be characterized by $c_{t+1}^T < c_t^T$, the model will forecast a real depreciation in period "$t+1$". Thus, the model predicts a depreciation coupled with *a contraction in private expenditure and a reduction in GDP in the same period.* There is mixed evidence with respect to the contemporaneous effect of the exchange rate on output as can be seen in for instance Brussiere *et al.* (2010).

IV. Searching for Wally

As explained above, our goal is identify the ex post consequences on sudden stops as a way to infer which are the macroeconomic conditions associated with those episodes. Our model predicts a positive relationship between nominal (current) consumption and income without liquidity constraints. Therefore, we are looking at a discrete and significant fall in consumption measured on in (constant) dollar terms.

We understand that SS are phenomena that can be experienced by almost any country, so we use a panel data for 34 countries for which reliable data are available. Our database gathers annual data for those countries for the period 1970-2016. The variables collected include current account in (current) dollars and percentage of GDP; consumption and GDP in (constant) dollars; and real multilateral exchange rates. The sources are the World Bank, the Economic Commission for Latin America and the Caribbean, the REER database of Darvas (2012), and data asked to several national statistical institutes.

The presumption that countries face SS when CA deficits are persistent implies that there should be a threshold when additional future CA deficit cannot be financed anymore. The identification of SS would thus correspond to a different configuration of accumulated CA deficits, and it is associated with a given drop in consumption and a current account reversal.

Let CA_t be the current account surplus or deficit at *t*, as a percentage of Y_t. We hypothesize that the SS is associated with the magnitude of CA_t deficit and its persistence. As such we evaluate ΔlnC_{t+1} for different configurations of (γ, h), where γ correspond to different CA_t threshold magnitudes, i.e., $CA_t < -\gamma$, and h to the number of periods for which the current account deficit was below that given value, so $CA_t < -\gamma, CA_{t-1} < -\gamma, CA_{t-2} < -\gamma, \ldots., CA_{t-h+1} < -\gamma$. We then define dummy variables for whether each observation of country *i* at period *t* satisfy the above condition, defined as $H_t(\gamma, h)$.

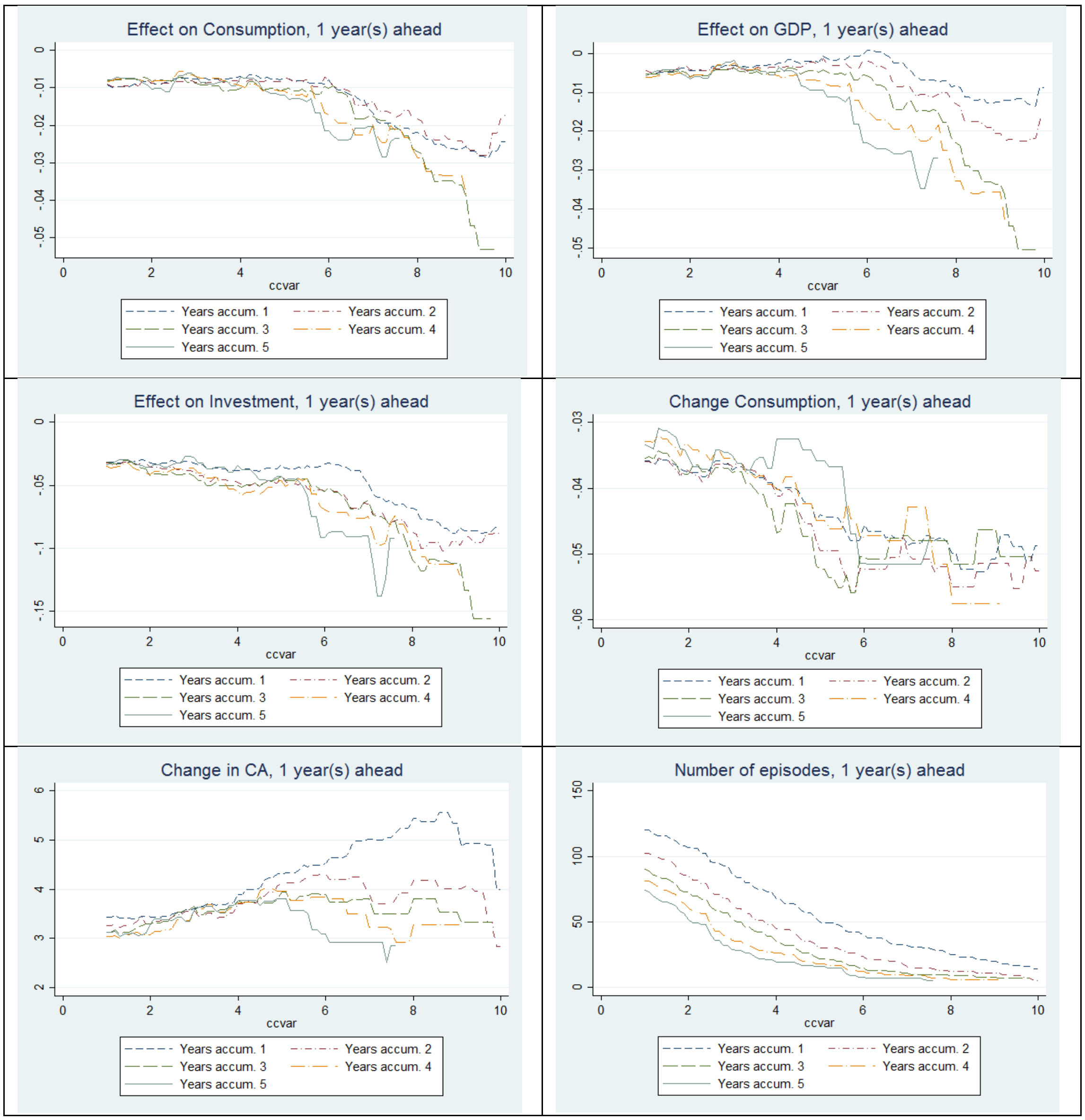
Now we compute the differences in consumption dynamics for countries with a particular configuration of current account deficits with $H(\gamma, h) = 1$ (the "treatment" group), with those that with $H(\gamma, h) = 0$ (the "control" group):

$$\mu_1(\gamma, h) = E[\Delta lnC_{t+1} | H(\gamma, h) = 1] - E[\Delta lnC_{t+1} | H(\gamma, h) = 0],$$

for $h = 1,2,3,4,5$ and for $\gamma \in \{1, 1.1, 1.2, \ldots., 10\}$ for a dense grid of 0.1% intervals in CA_t. $\mu_{1,t+1}(\gamma, h)$ thus corresponds to a difference-in-differences estimator, comparing the change in consumption of countries satisfying the H condition with respect to those countries that do not.

Figure 1-a plots the estimated effects (Figures 1-b and 1-c plot the same estimated effects for income and investment). By looking at the figures we can hypothesize on the corresponding CA deficits that are associated with SS. For instance, if we take the 5 years accumulated CA deficit curve (the solid line in panel a), there is a jump in the figure at about 6% CA deficit. While we are specifically interested in consumption behavior, the adjustment seems robust, since the same pattern is observed for income and income with similar drops (see panels b and c).

Figure 1



Now we impose two more conditions on the identified treatment group. If a SS occurs, we expect that the current account experiences a reverse and consumption drops. So now we estimate:

$$\mu_2(\gamma, h) = E[\Delta lnC_{t+1} \mid H(\gamma, h) = 1 \& \ E\Delta CA_{t+1} > 0 \ \& \ \Delta lnC_{t+1} < 0]$$

and

$$\mu_3(\gamma,h) = E[\Delta CA_{t+1}|H(\gamma,h) = 1\& \, E\Delta CA_{t+1} > 0 \,\&\, \Delta lnC_{t+1} < 0].$$

$\mu_2(\gamma,h)$ corresponds to the actual consumption drop that we estimate that is associated with a SS. $\mu_3(\gamma,h)$ corresponds to the current account reversal that should be observed after a SS. Figures 1-d and 1-e show the results. Figure 1-f plots the number of episodes that satisfy the following condition, i.e., $H(\gamma,h) = 1 \,\&\, \Delta CA_{t+1} > 0 \,\&\, \Delta lnC_{t+1} < 0$.

As we hypothesize above, our "Wally" must be hidden in a clear discontinuity of $\mu_1(\gamma,h)$ in the direction of γ for different fixed values of h. To avoid being deceived by mere visual inspection, we try to identify it by looking at the maximum numerical derivatives of the form $\mu_1(\gamma,h) - \mu_1(\gamma-\varepsilon,h)|\varepsilon$ for $\varepsilon = 0.1, 0.2, 0.3, 0.4{,}0.5$. Table 1 reports the 3 maximum numerical derivatives found. For each case we also report $\mu_2(\gamma,h)$ and $\mu_3(\gamma,h)$. In sum, the identification of a SS thus corresponds to the maximum drop observed in the derivative of $\mu_1(\gamma,h)$.

Table 1 aims to show that countries that experienced a current account deficit of 7% or more during 3 years will suffer a sudden stop measured by a 4.7% to 5.0% consumption drop, and a 4.0% current account reversal. Results for 2 consecutive years (not shown) are somewhat unstable, but show a simmilar picture. Table 2 and 3 identifies countries that experienced a current account deficit of 6% of the GDP or more during 4 to 5 years respectively will suffer a sudden consumption drop ranging between 4.4% and 4.9%, and a current account reversal between 3.2 and 3.8%. That's our guess of the place were Wally has to be hidden.

Table 1: Maximum numerical derivatives - 3 years accumulated of CA deficit

ε	Rank	Slope	γ	h	ΔlnC(t+1)	ΔCA(t+1)	# Episodes
0.1	1	-0,080	9.2	3	-0,0505	3,3	7
	2	-0,063	9.4	3	-0,0505	3,3	7
	3	-0,042	6.6	3	-0,0476	3,8	12
0.2	1	-0,032	9.4	3	-0,0505	3,3	7
	2	-0,025	9.2	3	-0,0505	3,3	7
	3	-0,018	8.2	3	-0,0515	3,8	9
0.3	1	-0,048	9.4	3	-0,0505	3,3	7
	2	-0,025	8.4	3	-0,0515	3,8	9
	3	-0,021	9.6	3	-0,0505	3,3	7
0.4	1	-0,068	9.4	3	-0,0505	3,3	7
	2	-0,048	9.6	3	-0,0505	3,3	7
	3	-0,038	8.4	3	-0,0515	3,8	9
0.5	1	-0,092	9.6	3	-0,0505	3,3	7
	2	-0,084	9.4	3	-0,0505	3,3	7
	3	-0,053	8.4	3	-0,0515	3,8	9

Notes: Identification of SS using maximum derivatives of $\mu_1(\gamma,h)$ in the direction of γ.

Table 2: Maximum numerical derivatives - 4 years accumulated of CA deficit

ε	Rank	Slope	γ	h	ΔlnC(t+1)	ΔCA(t+1)	# Episodes
0.1	1	-0,0532	9.1	4	-0,0575	3,3	6
	2	-0,0372	8.2	4	-0,0575	3,3	6
	3	-0,0357	5.7	4	-0,0447	3,8	13
0.2	1	-0,0322	5.7	4	-0,0447	3,8	13
	2	-0,0310	7.1	4	-0,0428	3,2	9
	3	-0,0277	7.5	4	-0,0472	3,2	8
0.3	1	-0,0334	5.7	4	-0,0447	3,8	13
	2	-0,0331	7.5	4	-0,0472	3,2	8
	3	-0,0310	7.1	4	-0,0428	3,2	9
0.4	1	-0,0424	5.9	4	-0,0475	3,8	12
	2	-0,0367	7.7	4	-0,0515	2,9	7
	3	-0,0360	5.7	4	-0,0447	3,8	13
0.5	1	-0,0740	5.9	4	-0,0475	3,8	12
	2	-0,0650	7.7	4	-0,0515	2,9	7
	3	-0,0567	7.9	4	-0,0515	2,9	7

Table 3: Maximum numerical derivatives - 5 years accumulated of CA deficit

ε	Rank	Slope	γ	h	ΔlnC(t+1)	ΔCA(t+1)	# Episodes
0.1	1	-0,0497	7.1	5	-0,0515	2,9	7
	2	-0,0477	5.9	5	-0,0514	3,1	8
	3	-0,0395	5.7	5	-0,0469	3,2	9
0.2	1	-0,0248	7.1	5	-0,0515	2,9	7
	2	-0,0248	5.7	5	-0,0469	3,2	9
	3	-0,0238	5.9	5	-0,0514	3,1	8
0.3	1	-0,0291	5.9	5	-0,0514	3,1	8
	2	-0,0236	7.5	5	-0,0476	2,9	5
	3	-0,0221	5.7	5	-0,0469	3,2	9
0.4	1	-0,0441	5.9	5	-0,0514	3,1	8
	2	-0,0390	6.1	5	-0,0515	2,9	7
	3	-0,0221	5.7	5	-0,0469	3,2	9
0.5	1	-0,0677	6.1	5	-0,0515	2,9	7
	2	-0,0664	5.9	5	-0,0514	3,1	8
	3	-0,0327	7.3	5	-0,0515	2,9	7

Notes: Identification of SS using maximum derivatives of $\mu_1(\gamma, h)$ in the direction of γ.

V. Summary

The goal of this paper was to show the connection between a sudden stop and debt issuance per year. Using a novel database we find that countries that experienced a current account deficit of 7% or more during 2 to 3 years will suffer a sudden stop measured by a 4.7% to 5.0% consumption drop and a 4.0% current account reversal. Similarly, countries that experienced a current account deficit of 6% of the GDP or more during 4 to 5 years will suffer a sudden consumption drop ranging between 4.4% and 4.9% and a current account reversal between 3.2 and 3.8%. In principle, these findings are the "Wally" we were looking for, and can be used as a leading indicator for this type of events.

Finding Wally, however, was not just an empirical quest. Using a novel recursive equilibrium notion due to Pierri and Reffet (2018) we were able to match these events and its properties by using a simple model without imposing shocks to deep parameters or additional structure to exogenous variables, as it is sometimes done in the literature. Our method, then, captures completely the multiplicity of equilibria latent in the sequential equilibrium and provides evidence in favor of interpreting a sudden stop as a coordination event, similar to a bank run.

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