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Abstract

In this article, we consider a nonlinear continuous-time model with discrete delays and externalities. We study the dynamic competitors's behavior, when they are heterogeneous in determining the output decisions. We have determined a unique positive equilibrium solution and we have applied mathematical techniques (Nyquist criterion) to estimate the range of the delays for which the equilibrium state remains asyntotically stable.
Keywords: Differential Delay Equations, Cournot model, Externalities, Equilibrium state.
JEL codes: C0

1 Introduction

Since the seminal work of Cournot [1], a huge numbers of researchers have been interesting in oligopoly models and their extensions.
At first, the most models assumed that each firm has instantaneous information about its output and also on the output of the competitors. This assumption is unrealistic in real economics, since there are always time delays due to determining and implementing decision. In this preliminary work, we assume there is fixed time delay in the firmss own output, the firms have got different decision mechanisms, namely a gradient rule and a best-reply rule through which the competitors adjust production (Pecora, et al, 2018) and an externality in the cost function has been added.

2 The model

We consider a duopolistic market in which the firms produce perfect sustitute goods. Let x and y represent the output of the two duopolists, X and Y, respectively. The demand function is linear, given by $p = a - b(x + y)$, $a, b > 0$. The cost functions are: $C_1(x, y) = c_1 x E(y)$ for X and $C_2(x, y) = c_2 y$ for Y, where $E(y)$ is a given externality, $c_1 > 0, c_2 > 0$.
From the hypothesis on demand and cost functions, the marginal profits of the firms are given by:

$$\phi_1(x, y) := -bx + (a - b(x + y)) - c_1 E(y)$$
$$\phi_2(x, y) := -by + (a - b(x + y)) - c_2.$$

Because firms haven't got a full knowledge of the concurrent's output, they are not able to compute the output that maximizes their profit, but they adopt a heuristic decision. In this work, we consider heterogeneous firms, each of one adopt different adjustment mechanism to set its output decisions. In particular, we following the framework introduced by [5] . We suppose one firm follows a myopic adjustment mechanism, while the other is partially adaptive. The so called myopic adjustment mechanism in discret time, read as:

$$y(t + 1) - y(t) = \alpha y(t)\phi_2(x(t), y(t))$$

where $\alpha > 0$ represents the speed of adjustment.
For the other mechanism, the naive firm expects the production of her rival doesn't inmediately jump to its new optimun quantity and then, it expects that the production the next period has not been changed, $(y^e(t+1) = y(t))$; in discret time it traslates into:

$$x(t + 1) - x(t) = \frac{a - by - c_1 E(y)}{2b} - x(t)$$

In this work, we deal with continuous-time, futhermore, we assume that firms have got a discrete lags in taking information about the levels in the production.
Finally, the temporal dynamics of the two-dimensional system of delayed differential equations are defined as:

$$x^{\cdot}(t) = \theta\left(\frac{a - by(t-\tau_1) - c_1E(y(t-\tau_1)) - 2bx(t-\tau_1)}{2b}\right)$$

$$y^{\cdot}(t) = \alpha y(t)\left[a - 2by(t-\tau_2) - bx(t-\tau_2) - c_2\right]$$
$$x(0^-) = 0,\ x(0^+) = k_1 > 0,\ y(0^-) = 0,\ y(0^+) = k_2 > 0,\ 0 < \theta < 1 \quad (1)$$

2.1 Equilibriun points and local stability

In order to search the equilibrium points of system (2),we consider the equations: $x^{\cdot} = 0, y^{\cdot} = 0$

Proposition 2.1 *Let* $a-2c_2 > 0, E \in C^1([0,\infty))$*, with* $E(0) > \frac{-a+2c_2}{c_1}, E^{'}(y) < \frac{3b}{c_1}$*, there exists a unique positive equilibrium point* (x^*, y^*) *of problem (2) iff* $y^* < \frac{a-c_2}{b}$.

Proof: Setting the condition $x^{\cdot} = 0, y^{\cdot} = 0$ in (2), we obtain the following equations system:

$$\frac{a-by^*-c_1E(y^*)-2bx^*}{2b} = 0$$
$$y^*[a - 2by^* - bx^* - c_2] = 0$$

and a simple computation shows that it is only solves by $(x^*, y^*) = (\frac{a-by^*-c_1E(y^*)}{2b}, y^*)$, where y^* is the unique solution of equation $3bx - c_1E(x) = a - 2c_2$.

2.2 Stability of equilibria

In order to deal with the local stability of the equilibrium positive state, we linearize the system (2) around the point (x^*, y^*) and we get (2):

$$\begin{pmatrix} x^{\cdot} \\ y^{\cdot} \end{pmatrix} = \begin{pmatrix} -\theta & \theta(-b - c_1E^{'}(y^*)2b) \\ -\alpha y^*(a - by^* - c_1E(y^*)2) & \alpha(a - 7by^* + c_1E(y^*) - 2c_22) \end{pmatrix} \begin{pmatrix} x - x(t-\tau_1) \\ y - y(t-\tau_2) \end{pmatrix} \quad (2)$$

We seek a solution of (2) of the form $u(t) = a_1e^{\lambda t}, v(t) = a_2e^{\lambda t}$. For non trivial solutions, this leads to the characteristic equation in λ,

$$\lambda^2 + \lambda\theta e^{-\tau_1\lambda} + d\lambda e^{-\tau_2\lambda} + re^{-(\tau_1+\tau_2)\lambda} = 0 \quad (3)$$

where

$$d = -\alpha(a - 7by^* + 2c_1E(y^*) - 2c_22)$$

and

$$r = -\theta\alpha(a - 7by^* + 2c_1E(y^*) - 2c_22) + \theta\alpha y^*(a - by^* - c_1E(y^*)4b)(-b - c_1E^{'}(y^*))$$

Remark 2.2 *We note that $\lambda = 0$ is a root of (3) if and only if $r = 0$*

2.2.1 Let $\tau_1 = 0, \tau_2 = 0$

In absence of delays, (x^*, y^*) is locally asymptotically stable if $\theta + d > 0$ and $r > 0$. A simple computation shows that $d > 0$, then we assume that $r > 0$. By remark 2.2, this implies that $\lambda = 0$ is not a root of (3) By continuity, for small $\tau_1 + \tau_2 > 0$, all eigenvalues of (3) have negative real parts provided that no eingenvalues bifurcate with real parts from $+\infty$ [2], then it is possible to use a criterion of Nyquist to estimate the range of $\tau_1 + \tau_2$ for whitch the equilibrium point remains asymptotically stable. Our main theorem is:

Theorem 2.3 *Let $d > 0$, $r > 0$ and $v_+ = \frac{1}{2}\left\{\theta + d + \sqrt{(\theta + d)^2 + 4r}\right\}$ There is a $T > 0$, such that if $\tau_1 \leq T$ and $\tau_2 \leq T$ implies that (x^*, y^*) is locally asymptotically stable.*

Proof We can choose $\mu > 0$, such that $\theta + d - \mu > 0$, then for $T > 0$ that satisfies the equation $\cos(Tv_+) = \frac{\mu}{\theta + d}$, we obtain $\tau_1 + \tau_2 < \frac{\mu}{r}$ and proof follows by [3].

2.3 Application

Let y is the output of a big firm Y, x is the output and $C(x) = c_1xE(y)$ the cost function of a fast food firm X. We can think that $\frac{\partial C}{\partial y}(x, y) < 0$ because if more employees of the firm Y consume in the firm X, the cost of firm X, as y dependent variable, will be lower, then $E^{'}(y) < 0$ and therefore $E(y)$ is in the condition of the proposition (2.1) and the theorem (2.3).

3 Conclusions

In this preliminary work, we have studied a dynamics continuous oligopoly Cournot model. Each firm has got a different adjustment mechanism, moreover because the tradings can occur at any time, we introduce different discrete lags in the production process and we have extended the paper [5] introducing an externality in the cost function. Under adecuate conditions, we have founded an unique positive equilibrium steady state and moreover, we prove it is asyntotically stable for a given bounded delays. In order to

study the occurrence of Hopf bifurcations (possible economics cycles) when time delays in production are different, a possible extension of the present work may be the use of analytical and geometrical results proposed by [4].

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