ASOCIACION ARGENTINA
DE ECONOMIA POLITICA

ANALES | ASOCIACION ARGENTINA DE ECONOMIA POLITICA

LIII Reunión Anual

Noviembre de 2018

ISSN 1852-0022
ISBN 978-987-28590-6-0

Constrained Efficient Entrepreneurship

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February 16, 2018

Abstract

In the standard incomplete markets (SIM) model the risk of low future labor income (e.g., unemployment) generates precautionary savings. This way, the relation between capital accumulation and consumption insurance is positive. This is not necessarily the case if agents face entrepreneurship choice. Entrepreneurs face risky investment and might be willing to trade insurance for accumulation depending on their permanent ability across sectors, idiosyncratic shocks, and wealth. We study the welfare properties of an entrepreneurship choice model using the notion of constrained efficiency where a utilitarian planner dictates occupational choices and savings across risk-free and risky assets, while respecting individual budget constraints. We find that although the planner increases aggregate capital, a result shared with the SIM models, the planner changes the composition of aggregate capital and favors the accumulation of the risky capital employed by entrepreneurs at the cost of insurance. The planner also runs into more debt to finance a higher amount of entrepreneurs, lowering the entrepreneurship productivity threshold of the decentralized economy. All these results are based on a two-period model that we are currently extending to an infinite horizon and realistically calibrated economy. Finally, we investigate policies that can bring the competitive equilibrium allocations closer to the constrained-efficient planner. The set of policies that we plan to investigate include (i) the capitalization of unemployment benefits, (ii) tax reductions for new businesses, and (iii) a "second" chance clause that allows for retakes of (i) and (ii) after previous exits from entrepreneurship (i.e., failure). An extension of the model to the possibility of selling businesses is on the way.

Contents

1 Introduction

We aim to study the welfare properties of a general equilibrium overlapping generations (OLG) model with workers and entrepreneurs, the former of which face idiosyncratic labor income risk and the latter idiosyncratic investment risk. In the standard incomplete markets (SIM) with only workers, consumption insurance and capital accumulation are positively related through precautionary savings (?). However, having entrepreneurs facing investment risk can change the sign of this relation if high-return projects, through accumulation of risky assets, are pursued at the cost of insurance ?. Our previous empirical work shows that this trade-off between accumulation/growth and insurance is quite relevant at the aggregate level (? and ?).

We argue that it is important to study a model with both salaried workers subject to labor income risk and entrepreneurs subject to investment risk that permits endogenous entry into entrepreneurship. Following ? (and henceforth DHKR), we evaluate the market outcome in such a model against the notion of constrained efficiency. Intuitively, constrained efficiency is achieved when the social planner can dictate agents' saving behaviors while respecting the individual budget constraints and marginal product pricing. Compared with the first best criterion used by ? or ?, constrained efficiency provides a straight-forward way to think about policies that can implement the constrained efficient outcome in that the social planner does not attempt to complete markets (Diamond, 1967).

As argued in DHKR, one of the key elements to quantitatively assess the difference between the competitive equilibrium allocations and constrained efficient allocations is the ability of the competitive setting to account for the wealth distribution. In their setting, the wealth distribution is matched with the help of an ad hoc earning income process. In our context, it is the endogenous entrepreneurial choice that helps generating the observed wealth dispersion in the US (?).

We propose to embed a model of entrepreneurial choice a la ? in the SIM model with uninsurable idiosyncratic labor income and investment shocks, and investigate the constrained

efficiency in this setting. The model can be calibrated to either the US economy or the Chinese economy, or both. We then use this framework to evaluate the performance of policies such as productivity-based investment loans and size-based investment loans and ask how much closer to constrained efficiency can these policies get the competitive outcomes.

There are three contributions our project will make to the existing literature. Firstly, our framework is the first in the literature of the SIM that combines labor income risk and investment risk, which is the necessary framework to quantify the trade-off between capital accumulation and consumption insurance at the aggregate level and investigate its welfare implications. Secondly, in relation to the literature on occupational choices and development, we provide a single framework where production risks, capital accumulation and consumption insurance are jointly determined and tied by the entrepreneurial choice. This is where we can discuss the effect from a given policy (such as a growth-enhancing policy of investment subsidies) on all three aspects of the macro-economy. In previous work, we highlight the significance of such a unified framework (?). Finally, as an effort to bridge the macro and micro literature, our OLG setting relates naturally to the voluminous literature on consumption where measures of consumption insurance over the life-cycle are developed in partial equilibrium settings.

The rest of the paper is sectioned as follows. In Section ??, we propose a two period model with occupational choice. We calibrate our model in Section ??. In Section ?? we present the results of our two period model and compares it with the SIM model, and also present exercises changing key parameters of the calibration. In Section ?? we perform our policy experiments. Section ?? concludes.

2 A Two-Period Model with Entrepreneurship: Some Useful Insights

We use a stripped-down two-period general equilibrium model with occupational choice and idiosyncratic income risks both for entrepreneurs and workers to illustrate the intuition. We study implications of this model for capital accumulation (the growth and level of aggregate capital) and insurance a la Townsend or some other statistic. Selection and misallocation implications are also discussed.

2.1 Competitive Equilibrium

There is a unit measure of agents heterogeneous in their permanent productivity as workers, y, permanent productivity as entrepreneurs, z, and in their initial wealth, ω. Suppose the triplet (y, z, ω) is distributed as $G(y, z, \omega)$ in the population. Agents have a time-separable CRRA utility function with discount factor β. There are two sectors, the corporate and the entrepreneurial sector. The corporate sector employs workers in efficiency units L, rents capital A from households, and produce according to a constant-return-to-scale production function, $F(A, L)$. The entrepreneurial sector produces at the unit of the household. Each entrepreneurial household has a decreasing-return-to-scale production function $f(z, k)$ that uses own physical capital k and his innate managerial ability z as inputs.

There are two periods. At the beginning of the first period, an agent indexed by (y, z, ω) chooses his occupation in the next period, whether to work in the corporate sector or to become an entrepreneur. If he chooses to become a worker in the corporate sector, then in the first period

he saves (or borrows) at the risk-free interest rate r and consumes, and in the second period he supplies inelastically labor, y in efficiency units, earns a wage income, earns the return on savings (or pay the debt), and consumes. The labor income in period two is risky. With probability half, he receives a positive shock e, and with the complementary probability, he receives a negative shock $-e$. He solves the following problem, taking wage w and interest rate r as given:

$$\begin{aligned} \max_{a} \quad & u(\omega - a) + \frac{1}{2}\beta \left[u(yw + e + ra) + u(yw - e + ra)\right] \\ & \text{subject to } a \geq -\frac{1}{r}\left(yw - e\right). \end{aligned} \tag{1}$$

Let the optimal saving be denoted a^* and the indirect utility function of a worker be denoted $U^w(y, \omega; w, r, e)$. It is easy to show that the optimal saving increases in the initial wealth ω and the labor income shock e, while decreases in the worker's productivity y.

If the agent chooses to become an entrepreneur, then in the first period he saves (or borrows) at the risk-free rate r, invests in physical capital k and consumes, and in the second period he produces using z and k, sells the output, earns the return on savings (or pay the debt), and consumes. The entrepreneurial output is likewise risky, with the size of the shock being ϵ. He solves the following problem taking the interest rate r as given:

$$\begin{aligned} \max_{a,k} \quad & u(\omega - a - k) + \frac{1}{2}\beta \left[u(zf(k) + \epsilon + ra) + u(zf(k) - \epsilon + ra)\right] \\ & \text{subject to } a \geq -\frac{1}{r}\left(af(k) - \epsilon\right); \\ & \qquad\qquad k \geq 0. \end{aligned} \tag{2}$$

Let the optimal savings and capital be denoted a^{**} and k^{**} and the indirect utility function of an entrepreneur be denoted $U^e(z, \omega; w, r, \epsilon)$. The FOCs imply that an entrepreneur always sets k^{**} to be such that $zf'(k^{**}) = r$. That is, he always operates at the optimal scale for his productivity z at the cost of capital r. Given z, entrepreneurs with lower initial wealth then necessarily hold

less (and possibly negative) safe asset:

$$\frac{da^{**}}{d\omega} = \frac{u"(c_1^{**})}{u"(c_1^{**}) + \frac{1}{2}\beta r^2 \left[u"(c_{2,H}^{**}) + u"(c_{2,L}^{**})\right]} > 0.$$

Coupled with higher risk in the entrepreneurial activity relative to the corporate sector, those with high entrepreneurial productivity and low initial wealth tend to be those facing the lowest consumption insurance. The loss in insurance might be so severe that makes entrepreneurship unattractive compared with employed work, even for someone who is highly productive in entrepreneurial activities.

The standard notion of competitive equilibrium applies:

Definition 1. *The competitive equilibrium is given by the occupational choice $o(y, z, \omega)$, holdings of the risk-free assets of the workers $a^*(y, z, \omega)$ and of the entrepreneurs $a^{**}(y, z, \omega)$, holdings of the physical capital of the entrepreneurs $k^{**}(y, z, \omega)$ such that*

1. *Given prices w and r, agents optimally self-select into the corporate or entrepreneurial sector, $o(y, z, \omega)$. For a worker, $a^*(y, z, \omega)$ solves (**??**). For an entrepreneur, $a^{**}(y, z, \omega)$ and $k^{**}(y, z, \omega)$ solve (**??**).*

2. *Labor and asset markets clear.*

To write down factors' market clearing conditions, we need first characterize the selection into entrepreneurship.

Proposition 2. *Suppose $\epsilon > e$. Increasing the initial wealth of an agent who is indifferent between becoming a worker and an entrepreneur makes him strictly prefer entrepreneurship.*

Proof. Use one star to denote the solution associated with worker's problem (**??**) and two stars to denote the solution associated with entrepreneur's problem (**??**). The proof proceeds

in three steps. In the first step, we show that the agent who values the consumption paths under the two occupation equally selects into the steeper consumption path, which entails lower consumption in the first period and higher expected consumption in the second period, upon an increase in his initial wealth. In the second step, we transform the entrepreneur's problem into an equivalent form to the worker's problem. In the final step, we show that $\epsilon > e$ implies that the entrepreneurship entails lower consumption in period one compared to the corporate sector, for a given level of life-time utility.

Step 1. The value of being a worker is then given by:

$$\begin{aligned}
&U^w = u(\omega - a^*) + \frac{1}{2}\beta\left[u(yw + e + ra^*) + u(yw - e + ra^*)\right] \\
\Rightarrow &\frac{dU^w}{d\omega} = u'(c_1^*)\left(1 - \frac{da^*}{d\omega}\right) + \frac{1}{2}\beta\left[u'(c_{2,H}^*) + u'(c_{2,L}^*)\right] r\frac{da^*}{d\omega} \\
&= u'(c_1^*).
\end{aligned}$$

The value of being an entrepreneur is:

$$\begin{aligned}
&U^e = u(\omega - a^{**} - k^{**}) + \frac{1}{2}\beta\left[u\left(zf(k^{**}) + \epsilon + ra^{**}\right) + u\left(zf(k^{**}) - \epsilon + ra^{**}\right)\right] \\
\Rightarrow &\frac{dU^e}{d\omega} = u'(c_1^{**})\left(1 - \frac{da^{**}}{d\omega} - \frac{dk^{**}}{d\omega}\right) \\
&+\frac{1}{2}\beta\left[u'(c_{2,H}^{**})\left(zf'(k^{**})\frac{dk^{**}}{d\omega} + r\frac{da^{**}}{d\omega}\right) + u'(c_{2,L}^{**})\left(zf'(k^{**})\frac{dk^{**}}{d\omega} + r\frac{da^{**}}{d\omega}\right)\right] \\
&= u'(c_1^{**}) - \frac{da^{**}}{d\omega}\left[u'(c_1^{**}) - \frac{1}{2}\beta\left(u'(c_{2,H}^{**}) + u'(c_{2,L}^{**})\right) r\right] \\
&-\frac{dk^{**}}{d\omega}\left[u'(c_1^{**}) - \frac{1}{2}\beta\left(u'(c_{2,H}^{**}) + u'(c_{2,L}^{**})\right) zf'(k^{**})\right] = u'(c_1^{**}).
\end{aligned}$$

The terms that disappear are due to the Envelope Theorem. Therefore,

$$\frac{d\left[U^w - U^e\right]}{d\omega} = u'(c_1^*) - u'(c_1^{**}).$$

Whether an increase in ω prompts the agent to become a worker or an entrepreneur depends

on the consumption paths under these two occupations.

Step 2. Rewrite the entrepreneur's problem (**??**) as follows. Let k^{**} be the solution to $zf'(k) = r$.

$$\begin{aligned} \max_{\kappa} \quad & u(\omega - \kappa) + \frac{1}{2}\beta \left[u(zf(k^{**}) - rk^{**} + \epsilon + r\kappa) + u(zf(k^{**}) - rk^{**} - \epsilon + r\kappa) \right] \\ & \text{subject to } \kappa \geq -\frac{1}{r}\left(zf(k^{**}) - rk^{**} - \epsilon \right). \end{aligned}$$

We can verify that $a** = \kappa - k^{**}$ and k^{**} satisfy the FOCs in the original entrepreneur's problem (**??**). Then this problem is isomorphic to the worker's problem with labor income $zf(k^{**}) - rk^{**}$ and labor income shock ϵ. In other words, the consumption path of an entrepreneur with productivity z and initial wealth ω is identical to the consumption path of a worker, who has the same initial wealth ω, an expected labor income $zf(k^{**}) - rk^{**}$ where k^{**} is the solution to $zf'(k) = r$, and faces a labor income shock of size ϵ.

Step 3. Consider a worker with productivity y, initial wealth ω and labor income shock e and an entrepreneur with productivity z, initial wealth ω and entrepreneurial shock ϵ. Due to the result in Step 2, the consumption path of the latter is identical to the consumption path of a worker with productivity $zf(k^{**}) - rk^{**}/w$, initial wealth ω and labor income shock ϵ. Therefore, a comparison of consumption paths between a worker and an entrepreneur can be conveniently reduced to a comparison of consumption paths between two workers with different labor income and labor income risks.

Consider a general form of the problem where the labor income is Y and the labor income shock is ε.

$$\begin{aligned} \max_{a} \quad & u(\omega - a) + \frac{1}{2}\beta \left[u(Y + \varepsilon + ra) + u(Y - \varepsilon + ra) \right] \\ & \text{subject to } a \geq -\frac{1}{r}\left(Y - \varepsilon \right). \end{aligned}$$

Let the optimal saving $a(Y,\varepsilon)$ be implicitly defined from the FOC of the above problem. Given $a(Y,\varepsilon)$, then the indifference curve $U^w(Y,\varepsilon,a(Y,\varepsilon)) = \overline{U}$ defines $Y(\varepsilon)$. That is, for each ε, there is a labor income level Y such that the indirect utility function evaluated at $(Y(\varepsilon),\varepsilon,a(Y(\varepsilon),\varepsilon))$ is exactly $\overline{U}$. We would like to study the change in a upon a change in ϵ, ensuring that Y adjusts to be on the indifference curve.

First, consider $\frac{dY}{d\varepsilon}$. Taking derivative with respect to ε on both sides of the identity:

$$u(\omega - a(Y(\varepsilon),\varepsilon)) + \frac{1}{2}\beta\left[u(Y(\varepsilon)+\varepsilon+ra(Y(\varepsilon),\varepsilon)) + u(Y(\varepsilon)-\varepsilon+ra(Y(\varepsilon),\varepsilon))\right] = \overline{U}.$$

yields

$$\begin{aligned}
& u'(c_1)\left(-\frac{\partial a}{\partial Y}\frac{dY}{d\varepsilon} - \frac{\partial a}{\partial \varepsilon}\right) + \frac{1}{2}\beta\left[u'(c_{2,H})\left(\frac{dY}{d\varepsilon} + 1 + r\frac{\partial a}{\partial Y}\frac{dY}{d\varepsilon} + r\frac{\partial a}{\partial \varepsilon}\right)\right. \\
& \qquad \left. + u'(c_{2,L})\left(\frac{dY}{d\varepsilon} - 1 + r\frac{\partial a}{\partial Y}\frac{dY}{d\varepsilon} + r\frac{\partial a}{\partial \varepsilon}\right)\right] = 0 \\
\Leftrightarrow \quad & \left[-u'(c_1) + \frac{1}{2}\beta\left(u'(c_{2,H}) + u'(c_{2,L})\right)r\right]\left(\frac{\partial a}{\partial Y}\frac{dY}{d\varepsilon} + \frac{\partial a}{\partial \varepsilon}\right) + \frac{1}{2}\beta\left(u'(c_{2,H}) - u'(c_{2,L})\right) \\
& \qquad + \frac{1}{2}\beta\left(u'(c_{2,H}) + u'(c_{2,L})\right)\frac{dY}{d\varepsilon} = 0 \\
\Leftrightarrow \quad & \frac{dY}{d\varepsilon} = -\frac{u'(c_{2,H}) - u'(c_{2,L})}{u'(c_{2,H}) + u'(c_{2,L})} > 0.
\end{aligned}$$

Next, consider $\frac{\partial a}{\partial \varepsilon}$ and $\frac{\partial a}{\partial Y}$ from the FOC of the worker's problem:

$$\begin{aligned}
\frac{\partial a}{\partial \varepsilon} &= -\frac{\frac{1}{2}\beta\left[u"(c_{2,H}) - u"(c_{2,L})\right]r}{u"(c_1) + \frac{1}{2}\beta\left[u"(c_{2,H}) + u"(c_{2,L})\right]r^2} > 0. \\
\frac{\partial a}{\partial Y} &= -\frac{\frac{1}{2}\beta\left[u"(c_{2,H}) + u"(c_{2,L})\right]r}{u"(c_1) + \frac{1}{2}\beta\left[u"(c_{2,H}) + u"(c_{2,L})\right]r^2} < 0.
\end{aligned}$$

Now consider the total differential in a, da, upon a change in ϵ and the induced change

in Y that keeps the life-time utility constant:

$$\begin{aligned}
da &= \frac{\partial a}{\partial \varepsilon} d\varepsilon + \frac{\partial a}{\partial Y} dY \\
&= \frac{\partial a}{\partial \varepsilon} d\varepsilon + \frac{\partial a}{\partial Y} \frac{dY}{d\varepsilon} d\varepsilon \\
&= -\frac{\frac{1}{2}\beta \left[u"(c_{2,H}) - u"(c_{2,L})\right] r}{u"(c_1) + \frac{1}{2}\beta \left[u"(c_{2,H}) + u"(c_{2,L})\right] r^2} d\varepsilon \\
&\quad + \frac{\frac{1}{2}\beta \left[u"(c_{2,H}) + u"(c_{2,L})\right] r}{u"(c_1) + \frac{1}{2}\beta \left[u"(c_{2,H}) + u"(c_{2,L})\right] r^2} \frac{u'(c_{2,H}) - u'(c_{2,L})}{u'(c_{2,H}) + u'(c_{2,L})} d\varepsilon \\
&= -\frac{\frac{1}{2}\beta r \left[u"(c_{2,H}) + u"(c_{2,L})\right]}{u"(c_1) + \frac{1}{2}\beta \left[u"(c_{2,H}) + u"(c_{2,L})\right] r^2} \left\{ \frac{u"(c_{2,H}) - u"(c_{2,L})}{u"(c_{2,H}) + u"(c_{2,L})} - \frac{u'(c_{2,H}) - u'(c_{2,L})}{u'(c_{2,H}) + u'(c_{2,L})} \right\} d\varepsilon
\end{aligned}$$

Focus on the terms in the bracket, since the utility function is CRRA, that expression is equivalent to

$$\frac{\left(\frac{c_{2,H}}{c_{2,L}}\right)^{-\sigma-1} - 1}{\left(\frac{c_{2,H}}{c_{2,L}}\right)^{-\sigma-1} + 1} - \frac{\left(\frac{c_{2,H}}{c_{2,L}}\right)^{-\sigma} - 1}{\left(\frac{c_{2,H}}{c_{2,L}}\right)^{-\sigma} + 1} = \frac{\left(\frac{c_{2,H}}{c_{2,L}}\right)^{-\sigma-1} - 1}{\left(\frac{c_{2,H}}{c_{2,L}}\right)^{-\sigma-1} + 1} - \frac{\left(\frac{c_{2,H}}{c_{2,L}}\right)^{-\sigma-1} \frac{c_{2,H}}{c_{2,L}} - 1}{\left(\frac{c_{2,H}}{c_{2,L}}\right)^{-\sigma-1} \frac{c_{2,H}}{c_{2,L}} + 1}$$

Note that the function $\phi(x) = \frac{\chi^{-\sigma-1}x-1}{\chi^{-\sigma-1}x+1}$ increases in x and therefore the expression above is negative. Hence, $\frac{da}{d\varepsilon} > 0$. This means, keeping the life-time utility constant, an increase in the labor income risk ε leads to an increase in savings. Increasing risk ε has two effects. The first effect is to increase the precautionary savings. The second effect works through increasing the labor income (Y) to compensate for the higher risk, which reduces savings. The net effect of both is positive on savings, as shown. Hence, the consumption path is steeper, with lower consumption in the first period, higher expected consumption in the second period and higher consumption variability across states in the second period, as one increases ε.

Then for an agent who is indifferent between being a worker and an entrepreneur, the fact that entrepreneurial risk is higher than labor income risk implies that the consumption path of an entrepreneur is steeper with lower consumption in the first period and higher expected

consumption as well as higher consumption volatility in the second period. Therefore, due to the result in Step 1, this agent will select into entrepreneurship if his initial wealth is increased. □

Proposition ?? implies that we can define cut-offs in initial wealth for each pair of productivities, $\hat{\omega}(y,z)$, above which agents become entrepreneurs. This makes the characterization of aggregate labor and capital supply straight-forward. The aggregate labor supply to the corporate sector becomes:

$$L = \int_{\omega \leq \widehat{\omega}(y,z)} y \, dG(y,z,\omega).$$

The aggregate capital supply to the corporate sector becomes:

$$A = \int_{\omega \leq \widehat{\omega}(y,z)} a^*(y,z,\omega) \, dG(y,z,\omega) + \int_{\omega \geq \widehat{\omega}(y,z)} a^{**}(y,z,\omega) \, dG(y,z,\omega).$$

It is evident that the cut-off $\widehat{\omega}(y,z)$ increases in y and decreases in z, since the indirect utility function of a worker (or an entrepreneur) increases in ω and y (or z). How do factor prices affect these cut-offs the define the occupation of an agent? We have the following corollary.

Corollary 3. *The wealth cut-off function in a competitive equilibrium with factor prices w and r, $\widehat{\omega}(y,z;w,r)$, has the following properties:*

1. $\frac{\partial \widehat{\omega}(y,z;w,r)}{\partial y} > 0.$
2. $\frac{\partial \widehat{\omega}(y,z;w,r)}{\partial z} < 0.$
3. $\frac{\partial \widehat{\omega}(y,z;w,r)}{\partial w} > 0.$
4. $\frac{\partial \widehat{\omega}(y,z;w,r)}{\partial r}$ *is ambiguous.*

Proposition 4. *The competitive equilibrium is generically not constrained efficient.*

Proof. The total differential of the indirect life-time utility of a worker gives

$$\begin{aligned} dU^w(y,\omega) &= \frac{1}{2}\beta\left[u'(c^*_{2,H}) + u'(c^*_{2,L})\right](ydw + a^*(y,\omega)dr) \\ &= \frac{1}{2}\beta\left[u'(c^*_{2,H}) + u'(c^*_{2,L})\right]\left[y\left(F_{LL}dL - \frac{A}{L}F_{AA}dA\right) + a^*(y,\omega)\left(F_{AA}dA - \frac{L}{A}F_{LL}dL\right)\right] \\ &= \frac{1}{2}\beta\left[u'(c^*_{2,H}) + u'(c^*_{2,L})\right]\left(\frac{y}{L} - \frac{a^*(y,\omega)}{A}\right)(LF_{LL}dL - AF_{AA}dA). \end{aligned}$$

Whether a worker would like an increase in the aggregate A or L depends on his factor endowments. If $\frac{a^*(y,\omega)}{y} < \frac{A}{L}$, then he prefers a higher A and lower L. If $\frac{a^*(y,\omega)}{y} > \frac{A}{L}$, then he prefers a lower A and higher L.

The total differential of the indirect life-time utility of an entrepreneur gives

$$\begin{aligned} dU^e(z,\omega) &= \frac{1}{2}\beta\left[u'(c^{**}_{2,H}) + u'(c^{**}_{2,L})\right]a^{**}(z,\omega)dr \\ &= \frac{1}{2}\beta\left[u'(c^{**}_{2,H}) + u'(c^{**}_{2,L})\right]a^{**}(z,\omega)\left(F_{AA}dA - \frac{L}{A}F_{LL}dL\right). \end{aligned}$$

Whether an entrepreneur would like an increase in the aggregate A or L depends on whether he is a saver or a borrower. If he saves in the safe asset, he prefers a lower A and higher L. If he borrows in the safe asset, he prefers a higher A and lower L. □

As the proof shows, workers with higher asset relative to his labor endowment gain from a lower A and a higher L and entrepreneurs with positive asset holdings (i.e. net savers) gain from a lower A and a higher L. This is intuitive. Both types of agents rely more on capital income than on labor/business income, and therefore would like to see an increase in the interest rate r by reducing the aggregate capital-to-labor ratio. In the next section, we solve formally for the constrained optimum.

2.2 Constrained Efficient Entrepreneurship

The notion of the constrained efficiency is the same as in **?**. The social planner chooses a wealth cut-off for each ability type, $\widehat{\omega}^P(y,z)$, above which agent-(y,z) becomes an entrepreneur, together with the saving function $a^{*P}(y,\omega)$ for the workers and the saving and investment functions $a^{**P}(z,\omega)$ and $k^{**P}(z,\omega)$ for the entrepreneurs to maximize a utilitarian social welfare function W:

$$W = \int_{\omega \leq \widehat{\omega}^P(y,z)} u(c_1^{*P}(y,\omega)) + \frac{1}{2}\beta \left[u(c_{2,H}^{*P}(y,\omega)) + u(c_{2,L}^{*P}(y,\omega))\right] dG(y,z,\omega)$$
$$+ \int_{\omega \geq \widehat{\omega}^P(y,z)} u(c_1^{**P}(z,\omega)) + \frac{1}{2}\beta \left[u(c_{2,H}^{**P}(z,\omega)) + u(c_{2,L}^{**P}(z,\omega))\right] dG(y,z,\omega),$$

where the social planner respects individual budget constraints and marginal pricing:

$$L^P = \int_{\omega \leq \widehat{\omega}^P(y,z)} y \, dG(y,z,\omega)$$
$$A^P = \int_{\omega \leq \widehat{\omega}^P(y,z)} a^{*P}(y,\omega) \, dG(y,z,\omega) + \int_{\omega \geq \widehat{\omega}^P(y,z)} a^{**P}(z,\omega) \, dG(y,z,\omega)$$
$$c_1^{*P}(y,\omega) = \omega - a^{*P}(y,\omega)$$
$$c_{2,H}^{*P}(y,\omega) = yF_L(A^P,L^P) + e + a^{*P}(y,\omega)F_A(A^P,L^P)$$
$$c_{2,L}^{*P}(y,\omega) = yF_L(A^P,L^P) - e + a^{*P}(y,\omega)F_A(A^P,L^P)$$
$$c_1^{**P}(z,\omega) = \omega - a^{**P}(z,\omega) - k^{**P}(z,\omega)$$
$$c_{2,H}^{**P}(z,\omega) = zf(k^{**P}) + \epsilon + a^{**P}(z,\omega)F_A(A^P,L^P)$$
$$c_{2,L}^{**P}(z,\omega) = zf(k^{**P}) - \epsilon + a^{**P}(z,\omega)F_A(A^P,L^P).$$

Table 1 Parameters

Parameter	Meaning	Value
β	Discount factor	0.98
δ	Depreciation in corporate sector	1
δ_e	Depreciation in entrepreneurial sector	1
α	Capital share corporate sector	0.33
α_e	Capital share entrepreneurial sector	0.33
η	Risk aversion	2
n_z	Number of managerial fixed effects	5
n_y	Number of working ability fixed effects	5
n_ω	Number of initial wealth per ability group	20
n_e	Number of transitory shocks to the working skill	2
n_ϵ	Number of transitory shocks to the managerial skill	2
$\mu_{z_{fix}}$	Mean fixed effect of managerial skill	1.24
$\sigma_{z_{fix}}$	Variance fixed effect of managerial skill	0.33
$\mu_{y_{fix}}$	Mean fixed effect of working skill	1.421
$\sigma_{y_{fix}}$	Variance fixed effect of working skill	0.425
$\rho_{y,z}$	Correlation between working and managerial fixed effect	1
$\rho_{a,z}$	Correlation between initial wealth and managerial fixed effect	0
$\mu_{e_{tran}}$	Mean transitory shock of working skill	0
$\sigma_{e_{tran}}$	Variance transitory shock of working skill	0.2
$\mu_{\epsilon_{tran}}$	Mean transitory shock of managerial skill	0
$\sigma_{\epsilon_{tran}}$	Variance transitory shock of managerial skill	0.5

3 Calibration

We populate the economy with joint densities of (y, z, ω). We study the case where (y, z) are positively correlated. We assume five productivity-z groups and five productivity-y groups. For the uncorrelated case, we assume wealth is uniformly distributed across z groups. In our benchmark case we assume no correlation between entrepreneurship productivity and initial wealth $\rho_{z,\omega}$. We will assess the cases of positive and negative signs for $\rho_{z,\omega}$ at the end of the Section **??**.

4 Quantitative Results

First, we explain the allocations of our competitive equilibrium. Second, we compare the competitive equilibrium against the allocations in the constrained efficient entrepreneurship problem. Third, we assess the differences between our results and those attained using the standard only-workers economy ?. Finally, we compare the benchmark economy to economies where we modified the relative risk of occupations (changing the worker risk keeping constant the entrepreneurial risk) and to economies where we change the initial distribution of wealth.

4.1 Competitive Equilibrium

Selection and Prices The percentage of entrepreneurs in the benchmark economy is 23% (column 2 in Table ??). The proportion of entrepreneurs increases with z: 0% of individuals in the lowest productivity group become entrepreneurs and this share reaches 10% in the median TFP group and 65% in the most productivity group (see Table ??). In terms of labor supply, the average productivity of workers is 0.20. The equilibrium implies an interest rate $r = 0.40$ and a wage rate $w = 0.608$.

Assets and Debts The share of risky capital in aggregate capital demanded by the entrepreneurs is 27.1%. The total amount of safe assets per worker is roughly three times larger than the total safe assets per entrepreneur (.88 against .33), and workers hold roughly 91% of the total safe assets. However, the total amount of assets per entrepreneur is almost twice as large as that of workers, 1.51, explained by the large amount of risky assets held by entrepreneurs, 1.21. The amount of risky assets per entrepreneur increases with z. On average, the lowest productivity group (with positive mass of entrepreneurs) has roughly 0.014 risky assets per capita, and the highest productivity group 0.18. Total savings are decreasing with z while debt is increasing with z. The risk premium is roughly one-third obtained by comparing the average marginal product of entrepreneurial capital of .48 versus an interest rate on the safe asset of .40. It is important

Table 2 Benchmark Results

	Model	
Variable	**CE**	**SP**
r	0.401	0.263
w	0.608	0.75
% E	23	62
z	0.480	0.414
y	0.202	-0.011
$K + A$	1.029	1.116
K	0.279	0.563
A	0.749	0.553
Y_2	1.302	1.355
$Var(log(c_2))$	0.124	0.155
U	-1.405	-1.392

to know that in CE there is no dispersion in the marginal product of entrepreneurial capital because the natural borrowing constraint (that ensures positive consumption tomorrow in the worst shock) does not bind as agents optimally choose future consumption to be positive.

Accumulation and Insurance The entrepreneurs hold more assets but a riskier portfolio (roughly 75% their assets are risky), while the workers hold roughly half of the assets that entrepreneurs hold with a portfolio entirely based on safe assets. That is, entrepreneurs accumulate more than workers, and they do so in riskier assets. This has implications for consumption insurance. The variance of consumption is larger for entrepreneurs than for workers, respectively, .096 and .078. This implies that entrepreneurs trade higher consumption levels (1.86 average consumption for entrepreneurs vs. 1.30 for workers) for lower consumption insurance and they are very willing to do so. Checking the average utility within each occupation, entrepreneurs do better than workers. That is, the accumulation versus consumption insurance trade off is present in our economy.

Table 3 Benchmark Results: Occupation Structure

	Model	
Variable	**CE**	**SP**
% **E**	**23**	**72**
% E (z_1)	0	0
% E (z_2)	10	50
% E (z_3)	10	70
% E (z_4)	30	90
% E (z_5)	65	100

Table 4 Benchmark Results: Asset and Debt Structure

	CE	SP
A	0.75	0.55
$Assets$	0.82	0.78
$Debt$	-0.07	-0.23
% Borrowers	20	31

Table 5 Benchmark Results: Asset and Debt Structure by Group

	A		Debt		Assets		% Borrowers	
Group	CE	SP	CE	SP	CE	SP	CE	SP
(z_1)	0.28	0.28	0	-0.000	0.276	0.284	0	5
(z_2)	0.23	0.19	-0.000	-0.004	0.227	0.192	5	10
(z_3)	0.19	0.11	-0.003	-0.018	0.189	0.126	10	35
(z_4)	0.10	0.02	-0.007	-0.077	0.110	0.098	15	50
(z_5)	-0.04	-0.05	-0.056	-0.13 0	0.015	0.082	70	55
Agrgegate	0.75	0.55	-0.066	-0.229	0.816	0.782	20	31

4.2 Constrained Efficient Entrepreneurship

Now, consider all the agents in the economy are my slaves. They choose a and k as I command. The agents have no rights. As enslaver (I am because I am going to tell some people to do things they don't want to do), I have two constraints, I have to respect your initial wealth (i.e., your budget constraint) and I also care equally about you all (Utilitarian social planner). Social planner here is an euphemism, this is not a guy that will achieve Pareto improvements.

Selection and Prices The planner increases the amount of entrepreneurs in the economy by increasing the amount of entrepreneurs in all the ability groups except the less able group. Entrepreneurship rates increases from 23% to 72%. The increase in entrepreneurship comes along with a decrease in the average quality of entrepreneurs (decreases from 0.48 to 0.40) since it is mostly the middle guys that SP switches from workers to entrepreneurs (see Table **??**). Since abilities are positively correlated, this increase in entrepreneurship brings a decrease not only in the number but also in the average quality of workers. The average quality of workers decreases from 0.20 to -0.12. With respect to prices, the wage rate w increases to 0.80 and the interest rate decreases to 0.23.

Assets and Debts For each and all TFP groups the amount of safe asset goes down (they run into debt), except for the first group who are always workers. Since now interest rates are lower, more people borrow, specially those that switch from workers into entrepreneurship. Even though the aggregate amount of risky capital increases the decrease in the aggregate safe asset more than compensates, leading to a decrease in the amount of assets in the economy. At the same time, the increase in risky capital is mostly finance by debt, since the groups that increase risky capital the most are also the ones that increase debt the most (see Table **??**).

Accumulation and Insurance Per capita output in the SP economy is larger than in the competitive economy, from 1.30 to 1.36. The larger output in the SP is accompanied by a larger amount of aggregate capital from 1.03 to 1.12, which is generated by a larger share of risky

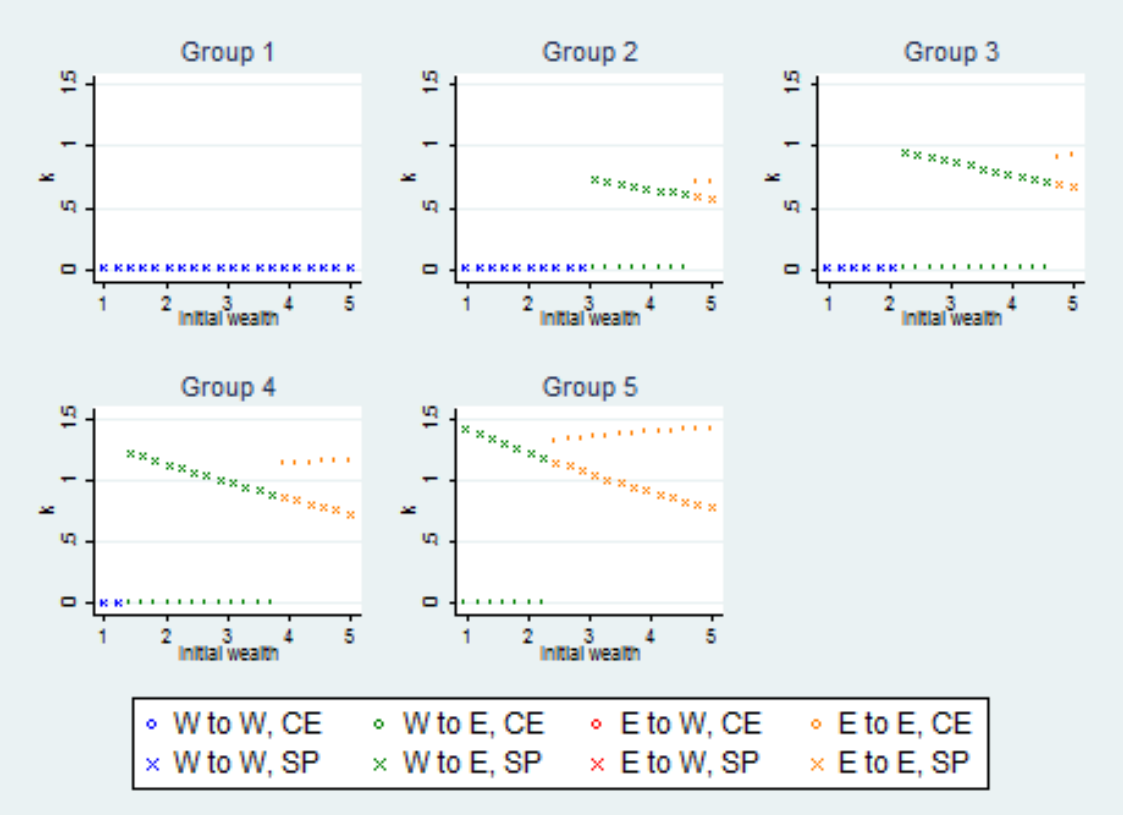
capital from 0.279/1.029=2% to.677/1.117=61%. Overall, the capital output ratio in the SP economy is similar to the CE. Given the similar capital to output ratio but a higher share of risky capital in the SP economy, we should expect worse insurance in the SP economy. This is what we find looking at the variance of C2 (.12 in CE and .17 in SP). The planner operates in an interesting way. Even though she increases the aggregate amount of risky capital, the allocation of this risky capital it is quite different than in the competitive equilibrium. The allocation of risky capital is decreasing in wealth in the SP solution (see Figure ??), meaning that wealth-poor agents have larger firms. However, the planner makes the wealth-rich agents to save more in the risk-free asset (see Figure ??). This contributes to face a decrease of the capital stock in the corporate sector much lower than the decrease in the labor supply. The capital to output ration increases from 0.83 to 1.45, which leads to an increase in wages and a decrease in the interest rates. These shifts in prices is what makes the redistribution to occur. Now wealth-poor agents who stay as workers are able to borrow cheaper and earn a higher wage in the second period, and at the same time, wealth-poor agents who become or stay as entrepreneurs are able also to borrow cheaper. All these effects together explains the winners and losers of moving to the SP economy. Rich agents who before were benefitting from high returns in their assets are now damage from the fall in the interest rate, while poor agents benefit from lower interest rates and higher wages (in the case of workers). See figure ??

4.3 Comparison with the only-workers economy

To generate an only-workers economy we close the entrepreneurial choice in our model, which brings us back to the SIM model ?.[1] Apart from this, the SIM economy that we study here is identical to our economy. In particular, we keep the same initial distribution of wealth, and the labor income process is the same across economies.

[1] An alternative would be to make the entrepreneurial option unattractive for self-interested individuals (e.g., by drastically dropping the permanent entrepreneurial productivity z for all groups). However, even under this alternative the social planner might optimally choose a non-zero mass of entrepreneurs.

Figure 1 Heterogenous Effects of Risky Capital

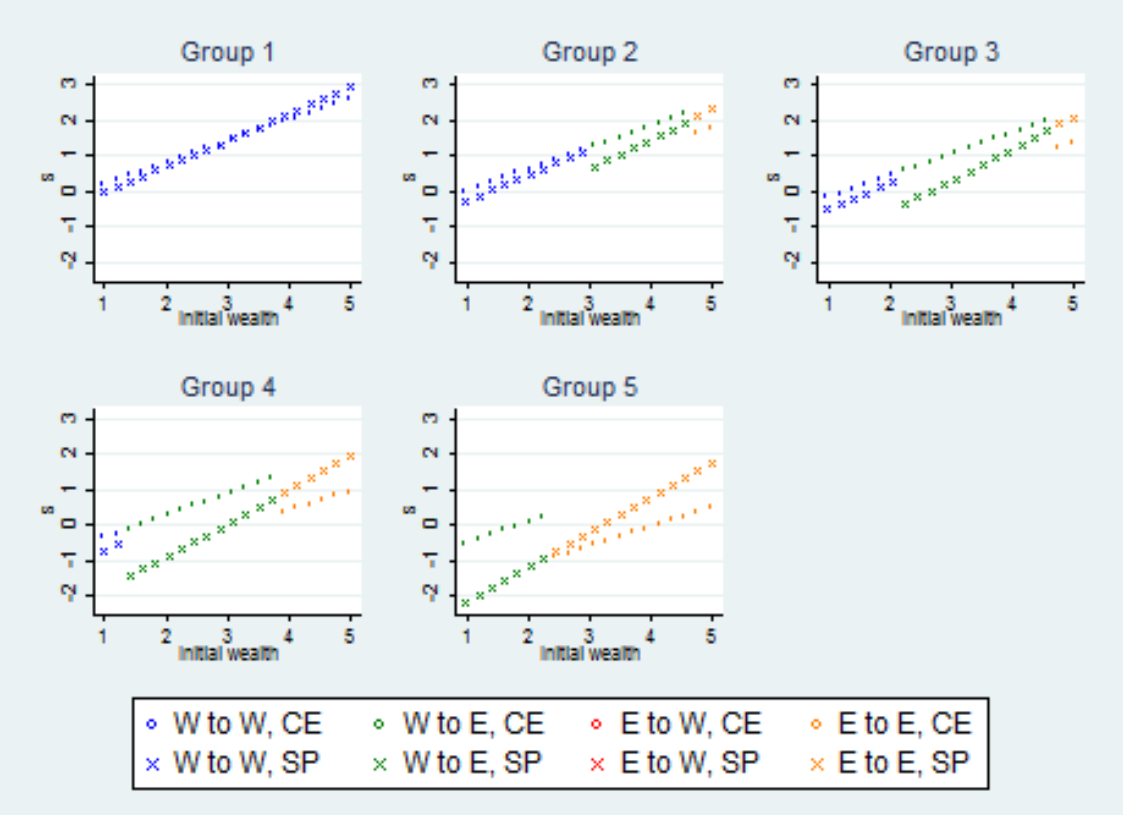


Selection and Prices In this setting everybody is a worker. Since in the CE entrepreneurs were selected from the most productive groups, and abilities are positively correlated these implies that average quality of workers is higher in the SIM economy (0.28 vs 0.20). The wage rate is a bit higher and the interest rate a bit lower than in the CE, however these differences are small.

Assets and Debts In this case all the assets is composed by the safe asset. However, even though there is only one type of assets, the aggregate amount of assets is larger than in the CE.

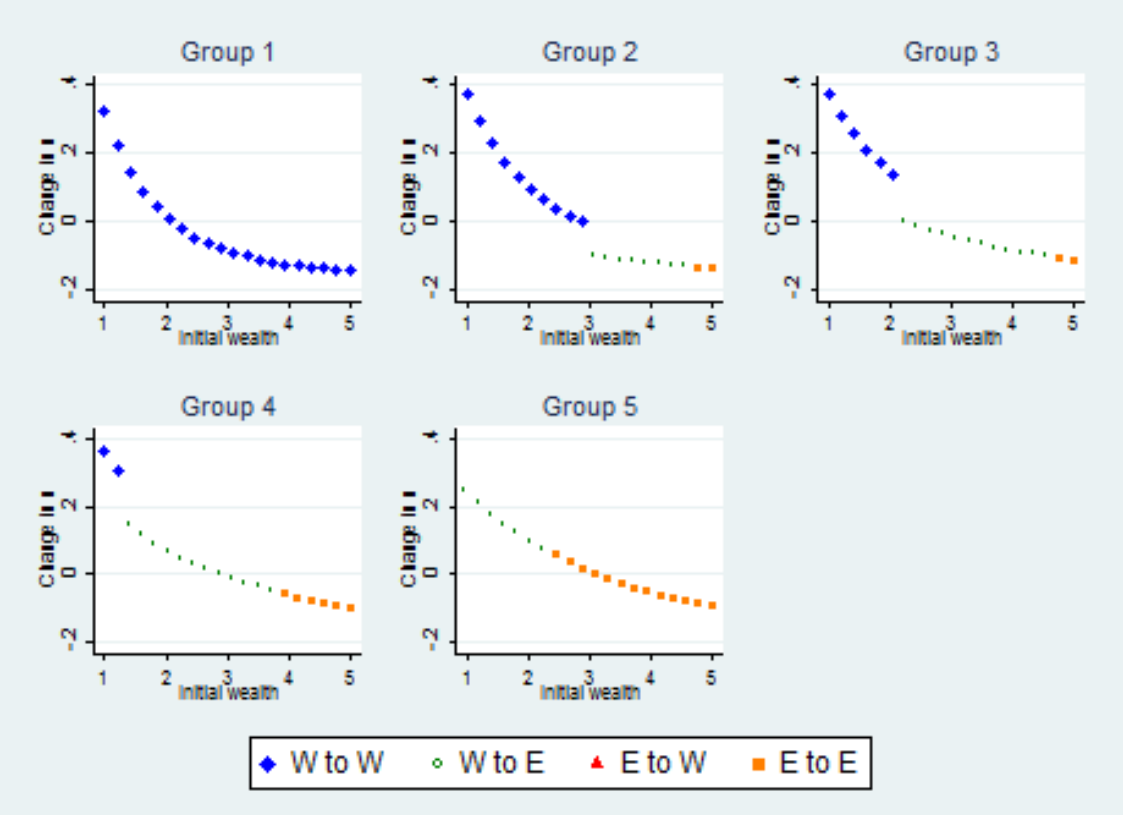
Accumulation and Insurance Per capita output (period 2) is larger in the benchmark economy. This is do to the entrepreneurial sector, since the corporate sector is larger in the SIM economy. This is because the output per entrepreneur in the entrepreneurial sector is much larger than the output per worker in the corporate sector. The larger increase in aggregate out-

Figure 2 Heterogenous Effects of Safe Capital



put in the benchmark economy makes the aggregate K/Y ratio larger in the SIM than in the benchmark case (0.82 and 0.79). Focusing on the corporate sector, K/Y is (hence the interest rate) is larger (smaller) in the benchmark economy than in the SIM economy. That is, the SIM economy favors borrowing and punishes a bit the savers. We find this is the case. Insurance is worse in the benchmark economy than in the SIM economy. We see this in the variance of consumption (period 2). Overall, people in the benchmark economy are better off than in the SIM economy, and also there is more room for the planner to improve the allocation.

Figure 3 Welfare Losers versus Winners



4.4 Changing Relative Occupation Risk

In this exercise we change the risk or workers occupation. In columns 2 and 3 (6 and 7) of Table ?? we decrease (increase) e from 0.2 to 0.1 (to 0.3).

Selection and Prices As we increase workers' risk the interest rate of the economy in the CE decreases from 0.406 to 0.393 and wages increase from 0.604 to 0.614. In the SP we observe similar patterns for interest rates and wages. What happens is that when we decrease (increase) workers' risk the entrepreneurial occupation becomes less (more) attractive than in the benchmark case, decreasing (increasing) the number of entrepreneurs from 23% to 17% (34%). This translates into more (less people) being worker pressuring the labor market to decrease (increase wages). So even though capital in the corporate sector increases (decreases), it is not

Table 6 Standard Incomplete Market (SIM) Economy Results

	SIM		Benchmark	
Variable	**CE**	**SP**	**CE**	**SP**
r	0.409	0.389	0.401	0.263
w	0.602	0.617	0.608	0.75
% E	0	0	23	62
z	-	-	0.480	0.414
y	0.286	0.286	0.202	-0.011
$A+K$	1.031	1.111	1.029	1.116
K	-	-	0.279	0.563
A	1.031	1.111	0.749	0.553
Y_2	1.264	1.296	1.302	1.355
$Var(log(c_2))$	0.096	0.102	0.124	0.155
U	-1.408	-1.406	-1.405	-1.392

sufficient to keep the wages at the level of the benchmark economy.

Assets and Debts Asset composition behaves as expected. Once we decrease (increase) workers' risk, the entrepreneurial occupation becomes relatively riskier (less risky) and as a result the accumulation of the risky capital decreases from 0.279 until 0.219 (0.397), and agents increase (decrease) their position in the safe asset A from 0.749 to 0.801 (0.643). This translates in a decrease (increase) of the total amount of assets in the economy. When we compare the planners choices in the three cases we get similar conclusions, with the interest fact that the total amount of assets (A+K) that the planner chooses in the three cases is the same, even though the composition is different. In any case, in the three cases the planner increases the amount of total assets in the economies, increasing the amount of risky capital, and decreasing the amount of safe capital.

Accumulation and Insurance Per capita output (period 2) is larger as we increase workers risk, but also insurance is lower. So as we increase the relative risk of being a worker, becoming an entrepreneur becomes more attractive, which translates, as we have discussed, in a larger proportion of investments in risky capital and larger output in period two. But this increase in

Table 7 Changing Relative Occupation Risk

	Decrease workers' risk		Benchmark		Increase workers' risk	
Variable	**CE**	**SP**	**CE**	**SP**	**CE**	**SP**
r	0.406	0.271	0.401	0.263	0.393	0.25
w	0.604	0.739	0.608	0.75	0.614	0.77
% E	17	58	23	62	34	68
z	0.213	0.021	0.480	0.414	0.171	-0.074
y	0.532	0.421	0.202	-0.011	0.442	0.405
$A+K$	1.020	1.116	1.029	1.116	1.041	1.116
K	0.219	0.511	0.279	0.563	0.397	0.648
A	0.801	0.605	0.749	0.553	0.643	0.469
Y_2	1.290	1.348	1.302	1.355	1.322	1.365
$Var(log(c_2))$	0.110	0.140	0.124	0.155	0.149	0.178
U	-1.397	-1.383	-1.405	-1.392	-1.418	-1.408

output is at the expenses of more volatile consumption in period two. Overall, the agents seem to prefer the economy with lower risk in the worker activity, which indicates that the increase in output is not sufficient to compensate the increase in consumption volatility that agents face.

4.5 Changing Initial Wealth Variance

In this exercise we change the initial distribution of wealth (ω) either by increasing the variance or by decreasing the variance keeping the mean, and so the total amount of initial wealth in the economy constant. In this three economies we will focus in the changes in the SP solution compare to the CE solution because interestingly, the CE solution for the three economies are very similar but the planner varies largely.

Selection and Prices While in the economy with lower initial wealth variance the planner increases the interest rate and decreases the wages in the benchmark case and in the higher initial wealth variance case, the planner do the opposite, decreasing interest rates and increasing wages. The planner reaches this solution by changing the occupation structure in the economy differently. While in the case with low variance the planner decreases the number of entrepreneurs in the

other 2 cases the planer largely increases the number of entrepreneurs. The increase (decrease) in the number of workers more than compensate the increase (decrease) in the amount of capital that the corporate sector uses (A) pushing wages down (up) and interest rates up (down).

Assets and Debts While the SP decreases the aggregate amount of assets (A+K) in the economy with low initial variance of wealth, she increases it in the other two economies. In the economy with low initial variance of wealth, the planner decreases the amount of risky capital and increases the amount of safe capital, but this increases is not sufficient to compensate the drop in K. In the other two economies, the planners doubles the amount of risky capital (K) and decreases the amount of safe asset (A), but not as much to compensate the increase in K. The number of lenders and borrowers in the economies also changes. While in the case of lower initial variance of wealth the planner slightly decreases the number of borrowers from 16% to 14% and slightly increases the number of lenders from 84% to 86%, in the other two economies the planner increases the number of borrowers and decreases the number of lenders (this effect also comes from the changes in the interest rates).

Accumulation and Insurance In the economy with low variance of initial wealth the planner decreases per capita output (period 2) and also decreases the volatility of consumption, so it gives more insurance at the expenses of growth. In the benchmark economy, as we have pointed out before the planner increases per capita output at the expenses of less insurance in period 2. However, once we increase the variance of initial wealth the planner is able to increase output and insurance in period 2. Comparing the utility changes we observe that as we increase the variance of initial wealth there is more room for the SP intervention. Also we observed that in cases were the initial distribution of assets present a larger variances the planner can achieve larger growth and more insurance.

Table 8 Change Initial Wealth Variance

	Decrease wealth variance		Benchmark		Increase wealth variance	
Variable	**CE**	**SP**	**CE**	**SP**	**CE**	**SP**
r	0.401	0.415	0.401	0.263	0.400	0.165
w	0.608	0.597	0.608	0.75	0.609	0.947
% E	19	14	23	62	28	81
z	0.199	0.220	0.480	0.414	0.199	-0.156
y	0.541	0.565	0.202	-0.011	0.442	0.351
$A+K$	1.029	1.009	1.029	1.116	1.029	1.178
K	0.248	0.204	0.279	0.563	0.326	0.708
A	0.781	0.806	0.749	0.553	0.703	0.470
Y_2	1.294	1.283	1.302	1.355	1.308	1.351
$Var(log(c_2))$	0.071	0.064	0.124	0.155	0.226	0.189
U	-1.336	-1.336	-1.405	-1.392	-1.553	-1.434

5 Policy Experiments

TBC

6 Conclusion

TBW

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A Solution algorithm

A.1 Two-Period Model

Competitive Equilibrium In order to solve the model we proceed as follows. First we create a grid for managerial ability z, working ability y, and initial wealth ω. We allow z and y to be correlated, and we also allow for correlation between z and ω. Each combination of abilities (y, z) will determine an ability group. For each ability group we assume that initial wealth is uniformly distributed between $a_{min}(y, z)$ and $a_{max}(y, z)$. Note that the limits depend on the abilities, allowing us to have different grids for different occupation groups. To the model presented before we make one change, which is to assume that risk is multiplicative and not additive. This means that labor income for a worker in the second period is $(y + e) * w$ and for an entrepreneur is $(z + \epsilon) * k^{\alpha}$. Once we have the grids constructed we can proceed to solve for the equilibrium:

1. Guess an interest rate. Given an interest rates we can compute the wage using the FOC of the corporate sector.
2. For each agent (z_i, y_i, ω_i) compute his utility as worker and as entrepreneur.
   (a) To obtain the optimal saving (a_i) when worker we use the FOC as a worker.
   (b) To obtain the optimal decisions (a_i, k_i) when entrepreneur we use the FOC as a entrepreneur (a system of 2 equations).
3. Choose the occupation that delivers the maximum utility.
4. After solving for all i, aggregate individual savings $\left(\sum_{i=1,N} = \mu_{a_i} * a_i \text{ where } \mu_{a_i} \text{ is the mass of people of type } i\right)$ and compute the excess demand for capital in the corporate sector (A), and update the guess if the tolerance is not satisfied.

Constrained Planner

1. Give a guess for the occupation cut-off for each -y,z- group. This cut-off is a wealth level above wich agents of the type -y,z- decides to become entrepreneurs. Given this cut-off we can compute the social planner occupational structure.
2. Now that we have the occupational structure we can give a guess for savings for workers, and savings and physical capital for entrepreneurs.
3. Given all the guesses (cut-offs, savings, and physical capital) we can compute aggregate labor (L) and capital (A) in the corporate sector, and then compute the marginal products at which the agents will be paid. Note that to compute prices we are using as capital the capital demand, and so we have aggregate consistency.
4. With all this information we can compute the social welfare value. We use an equation solver [2] to solve for the optimal savings and physical capital decisions that maximize the social planner welfare value for a given cut-off.

[2] fminunc is a solver that uses quasi-newton methods to minimize a function

5. Finally, we use the Nelder-Mead method to find the occupation cut-offs that maximize the social welfare function.