ECONOMIC NONEQUILIBRIUM, INSTABILITY AND CHAOS

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Introduction

In a former article[1], we have argued in favor of a broader use of nonlinear dynamical analysis in economic theory as a way of studying the processes of change in the economy.

We have stressed that this approach allows us to think change in a different way to the one prevalent in economic theory, where evolution is seen just as a smooth, gentle, continuous process.

On the contrary, nonlinearity paves the way to the analysis of economic discontinuity, i.e., abrupt, sharp changes in economic variables, like the October 1987 stock market crash, the currency crises that shattered the Bretton Woods system in the early '70s or hyperinflation processes.

We posit that this sort of phenomena are better analyzed within the framework provided by nonlinear dynamics.

In this paper, we illustrate by means of an oversimplistic example some implications of the adoption of that approach and we further analyze its consequences from a methodological point of view.

Dynamic instability in market analysis

As it is well known, under the assumption of linearity in the demand and supply functions, Walrasian dynamic stability depends on the slopes of the demand and supply curves as well as on the adjustment coefficient of prices with respect to excess demand.

Instability, in the linear case, implies an explosive time path. As in reality explosive time paths seem to be a somewhat rare and very particular case, if at all, it is usual to assume well-behaved demand and supply curves, i.e., curves that warrant stability of equilibrium prices.

Although linearity is recognized to be too restrictive an assumption, the wide use of the linear approach is justified from the point of view of *local* stability analysis.

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1 V.A. Beker (1994).

However, local stability is a narrow concept for economic applications. It restricts stability analysis to fluctuations that keep the system close to its equilibrium values.

Although the linear assumption is an acceptable approach for close-to-equilibrium analysis, this is not the case when the aim is *global* stability analysis, which is far more important from the point of view of applied economics.

As soon as we leave close-to-equilibrium analysis, linearity, in most cases, is no longer a reasonable assumption. For instance, in a linear unstable system a small perturbation will increase indefinitely. That is why, in business cycle theory, "ceilings" and "floors" were introduced (Hicks, 1950) as a way to put a limit to fluctuations. However, this in itself implies to resort to a nonlinear element (Blatt, 1983, p. 162).

As a general case, far-from-equilibrium analysis implies the use of nonlinear models.

Market stability analysis with nonlinear functions

We will illustrate the complexities that arise as soon as we abandon the linearity assumptions analizing a case of market stability. For that purpose we will employ a logistic equation model (Peters, 1991).

Although this model is extremely simplistic it is useful to illustrate the sort of complexities that arise in even a simple nonlinear system. We can begin to imagine the complex results that can originate in more realistic and thus larger nonlinear systems.

Let us suppose that the demand is represented by a logistic equation of the form:

$$D_t = \alpha P_t(c-P_t)$$

where D_t is the quantity demanded in period t and P_t is price in the same period. For the sake of convenience let us assume in what follows that c=1, i.e., P_t takes values in the interval [0,1]. Anyway, the qualitative results hold for any other value we want to give to c.

What does this demand function mean? As illustrated in Figure 1, as price begins increasing the quantity demanded increases too until reaches a maximum when P=0.50. Further increases in price are accompanied by a quantity decline as in well-behaved demand curves[2].

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[2] It is interesting to remark that one of the pre-Marshallian economists dealing with the laws of demand and supply, Hans von Mangoldt (1824-1868), cited the case of demand curves that rise with price because of expectations of even higher future prices. Contradicting his contemporary Dupuit -who argued that demand curves must be of convex shape- Mangoldt also held that negative sloping demand cuves could be either convex or concave depending on

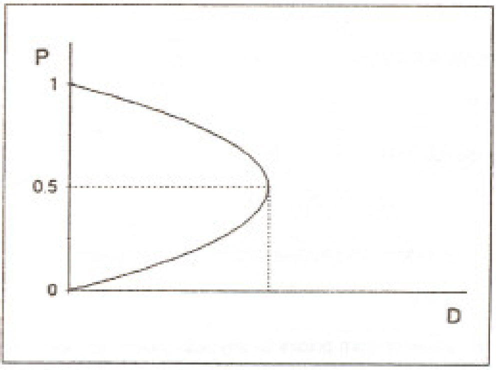

Fig. 1

An upward-sloping segment of the demand curve is a familiar feature in asset markets. As assets are demanded as value bearers, the more the price is expected to increase in the future the higher will be its demand today. So, when prices begin moving up, it is interpreted as an announcement of further increases and this stimulates the quantities demanded[3].

In a context of rational expectations, market prices are supposed to reflect all the available information. Therefore, they may be considered by the less-informed agents as revealing the information possessed by informed traders. Positive feedback trading -buying after price increases- may be, from this point of view, quite a rational behavior.

Positive feedback trading may also result from technical analysis models designed to catch incipient trends[4], from the use of stop loss orders, from portfolio insurance, from a positive wealth elasticity of demand for risky assets, or from margin call-induced selling after periods of low returns (Cutler et al, 1990).

Positive feedback trading may also be the result of herd behavior, i.e., investors driven by group psychology, simple mimicking the investment decisions of other investors.

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the type of goods (luxuries or necessities),on the degree of inequality of income distribution, and on the availability of close substitutes (Humphrey, 1992).

[3] This is a particular case of the more general one where price is taken as a sign of quality. See Scitovsky (1944-45) and Kreps (1992).I want to thank my son Pablo, who draw my attention on this point.

[4] With reference to the October 1987 crash Martin Feldstein states: "Institutional portfolio managers were blamed for program trading strategies that involved selling stock as equity prices fell" (M.Feldstein, 1991, p. 8). In the same volume, Lawrence H. Summers mentions The Economist as saying that "almost four-fifths of foreign exchange trading is driven by technical systems that give rise to positive feedback" (Ibid, p. 141).

Let us postulate a linear supply function

$$S_t = \beta P_t$$

The excess demand function is then:

$$E_t = D_t - S_t = \alpha P_t (1 - P_t) - \beta P_t$$

Let us assume a Walrasian mechanism for price adjustment:

$$P_{t+1} - P_t = k E_t$$

As β is just a converter from prices to physical quantities, we may assume

$$k = \frac{1}{\beta}$$

Thus

$$P_{t+1} - P_t = \frac{1}{\beta} [\alpha P_t (1 - P_t) - \beta P_t] \quad (1)$$

$$P_{t+1} - P_t = \frac{\alpha}{\beta} \quad P_t (1 - P_t) - P_t \quad (2)$$

Calling

$$\gamma = \frac{\alpha}{\beta}$$

it becomes

$$P_{t+1} = \gamma P_t (1 - P_t) \quad (3)$$

which is, again, a logistic equation.

Our interest is to know the asymptotic behavior of this difference equation.

For that purpose, let us take any initial value for P_t and any value for γ included in the interval $1<\gamma<3$. If we iterate equation (3) we will realize that prices always converge to a single value, whichever be the initial price. For instance, Figure 2 depicts the case for $P_0 = 0.30$ and $\gamma = 2$.

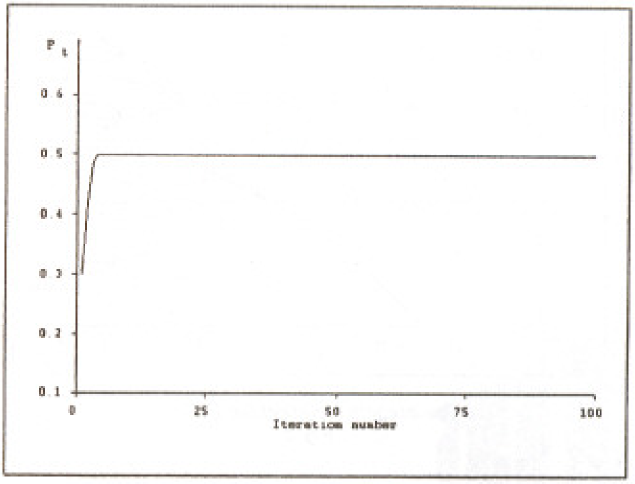

Fig. 2

This behavior suddenly changes when $\gamma=3$. At this *critical level* there appears a *bifurcation*. The system oscillates between two values. The same happens again when $\gamma=3{,}45$; then, four possible solutions appear.

The route to chaos

Broadly speaking, if $F_\gamma(x)=\gamma x(1-x)$, F_γ has an attracting fixed point at $P_\gamma=\frac{(\gamma-1)}{\gamma}$ provided[5] $1<\gamma<3$. As γ passes through 3, a bifurcation takes place: a new periodic point[6] of period 2 appears. As γ continues to increase the dynamics of F_γ becomes increasingly more complicated: it undergoes a series of period-doubling bifurcations (see Fig. 3). Finally, F_γ becomes chaotic.

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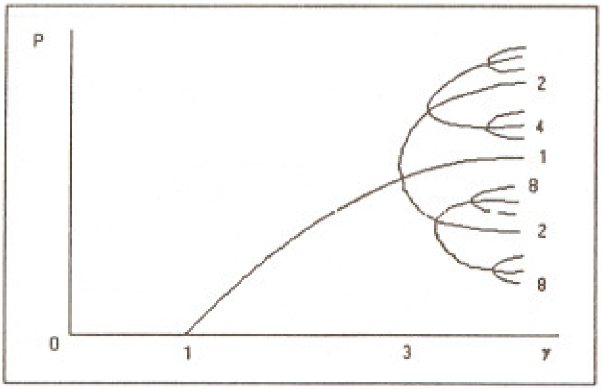
[5] If $\gamma\leq 1$, $p\leq 0$.

[6] The point x is a periodic point of period *n* if $f^n(x)=x$, where $f^n(x)=\underbrace{f\circ\cdots\circ f(x)}_{n\ \text{times}}$, that is the n-fold composition with itself.



(The integers represent the periods)
Fig. 3

In fact, a necessary and sufficient condition for the fixed point to be stable is that the absolute value of the derivative of F_γ at $x_0 = P_\gamma$ be less than 1; i.e., $\left|\frac{dF_\gamma}{dx_0}\right| < 1$. As the value of γ increases the hump of F_γ becomes higher and P_γ moves down into regions where the slope is greater. Thus the fixed point becomes unstable[7].

What does chaos mean?

Let V be a set. $f:V \to V$ is said to be chaotic on V if:

a) f has sensitive dependence on initial conditions;

b) f is topologically transitive[8].

This means that a chaotic map possesses two basic ingredients: unpredictability and indecomposability.

A map possesses *sensitive dependence on initial conditions* if there exist points arbitrarily close to a point x which eventually separate from x by at least a certain $\delta>0$ under iteration of f. This makes a chaotic map unpredictable.

More formally, $f:J \to J$ has sensitive dependence on initial conditions if there exists $\delta>0$ such that, for any $x \in J$ and any neighborhood N of x, there exists $y \in N$ and $n \geq 0$ such that $\left|f^n(x) - f^n(y)\right| > \delta$.

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[7] It can be easily proven that $\frac{dF}{dx} < 1$ implies $\gamma<3$.

[8] Some authors include a third condition: periodic points are dense in V. (See Devaney, 1989, p. 50).

A chaotic map cannot be broken down into two subsystems which do not interact under f. In fact, a *topologically transitive* map has points which eventually move under iteration from one arbitrarily small neighborhood to any other. Then, the dynamical system cannot be decomposed into two disjoint open sets which are invariant under the map.

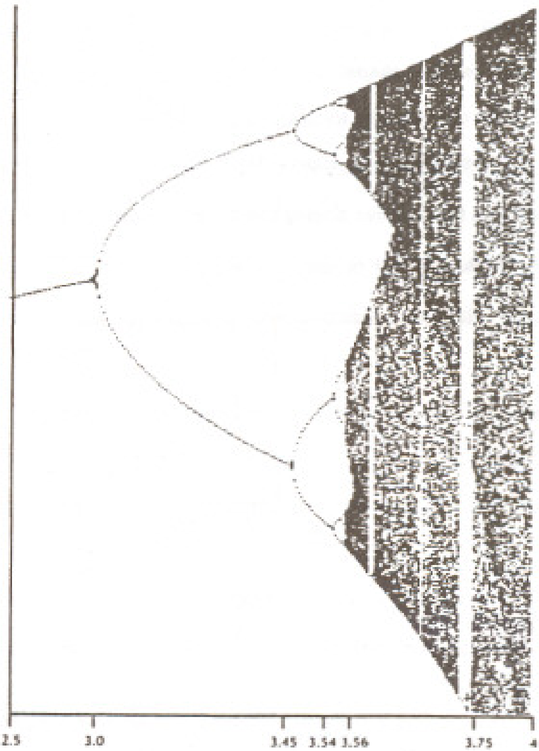

Fig. 4

More formally, $f{:}J \rightarrow J$ is topologically transitive if for any pair of open sets $U,W \subset J$ there exists $k>0$ such that $f^k(U) \cap W \neq 0$.

Indecomposability means, in essence, that any subset, whatever its size, gives way to different trajectories diverging under iteration. The opposite happens in the case of stable systems where neighboring points are transformed into neighboring points -or into one single point.

Turning back to our initial example, we have in Figure 4 the orbit diagram of F_γ with $2.5\leq \gamma \leq 4$. It plots potential values of x versus the associated values of γ.

As it can be observed, there appear succesive bifurcations until for each value of γ we have infinite solutions in the chaotic region. Although generated by a deterministic equation the series looks random.

Interpreting the genesis of chaos.

Whose is the responsibility for this change of behavior?

As we have seen, instability appears in the interval $3\leq \gamma \leq 4$. This means that unstable equilibrium occurs on the negative sloping branch of the demand curve of Figure 1 (that is,for $P_t>0.5$) (see Figure 5).

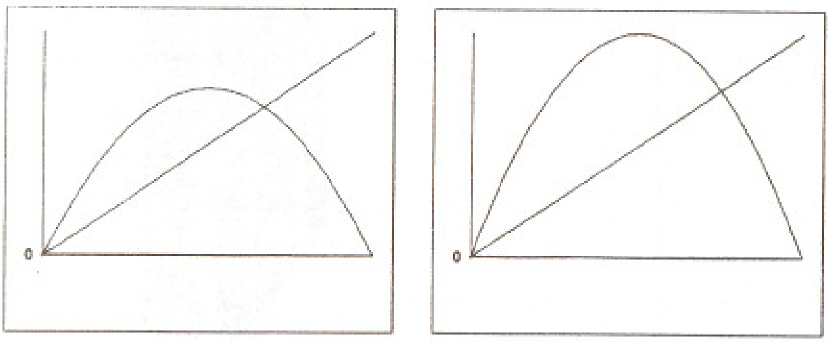

Fig.5. The graphs of F_γ $(P_t)=\gamma$ $P_t(1-P_t)$ for $\gamma=3$ and $\gamma=4$ from left to right. In both cases P_t^* >0.5.

In other words, a sufficient condition for equilibrium stability is for it to take place on the positive sloping branch of the demand curve.

On the other hand, a *necessary* condition for instability is that equilibrium takes place on the negative sloping branch of the demand curve. However, it is not a sufficient condition as it may be immediately verified for $2<\gamma<3$.

A *sufficient* condition for instability is $\gamma\geq 3$. The equivalence between this condition and the one in the case of a linear demand curve is demonstrated in the Appendix.

The aforementioned condition means that instability, in our model, is proper of a bull market.

Let us recall that $\gamma = \frac{\alpha}{\beta}$. So, the increases in the value of γ are explained by an increase in the value of α or a decrement in the value of β, or both.

α comes from the demand function:

$$D_t = \alpha P_t(1-P_t) = \alpha(P_t - P_t^2)$$

The term between brackets may be interpreted as a sort of "corrected" price. Then, α may be considered the adjustment coefficient of demand to changes in the "corrected" price. An increase in α moves the demand curve to the right in Figure 1.

β measures the slope of the supply curve and its inverse is the coefficient of reaction of price to excess demand.

So, as β decreases, the supply curve moves to the left, augmenting the excess demand at a given price, and, at the same time, it increases the price reaction to a given excess demand.

That is why we have said that instability is a characteristic of a bull market: it depends positively on increases in demand and in the coefficient of reaction of price as well as on decrements of supply. When, due to the combination of these elements, γ reaches the value of 3, instability appears on the stage.

Instability increases as γ tends to 4, which is the maximum value it can reach in our exercise. In that case the critical point for P_{t+1} equals 1, which by hypothesis, is the maximum value the price may take.

Where does instability come from?

When γ passes through 3 there appears an attracting periodic point of period 2. The system is attracted to it and enters a loop oscillating between two prices without converging to the equilibrium point[9] (see Figure 6).

[9] Strictly speaking, the set of all points which under iteration are periodically visited by the system, always in the same order, form a periodic orbit or limit cycle (see Medio, 1992,p. 45).

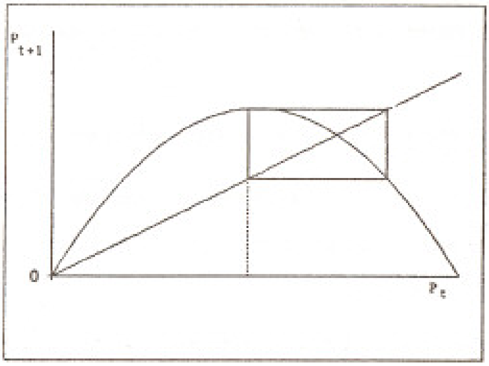

Fig. 6

As γ increases new attracting periodic points are born of period 4, 8, etc. F_γ undergoes a series of period-doublings as γ increases until it becomes chaotic.

Thus, while the linear approach allows, in general, only two alternatives: either stability or an explosive path, nonlinearity allows for a continuum of alternatives from stable equilibrium up to chaos, depending on the value of the parameters.

The measurement of chaos

It is still an unsettled question whether strange attractors[10] are really the key to chaos or not[11]. Some authors argue that chaotic dynamics depend on the existence of a strange attractor. However, Eckmann and Ruelle (1985) have argued that it is sensitive dependence on initial conditions which is the true meaning of chaos, on the ground that the dynamical aspect is a more important aspect than the geometrical one.

Anyway, inasmuch as sensible dependence on initial conditions is the essential feature of chaotic dynamics the measure of chaos is provided by the Lyapunov exponent, more precisely by the largest Lyapunov exponent.

Lyapunov exponents measure how quickly nearby orbits diverge in phase space. Thus, they measure the susceptibility of a system to sensitive dependence on initial conditions.

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[10] Strange attractors are a kind of attractors which, unlike point attractors or limit cycles, are nonperiodic but whose points and orbits stay within the same region of phase space. For a nice analysis of strange attractors see Medio (1992).

[11] One of the reasons for this is that there is no agreement on what a strange attractor is. There is no consensus at all on the use of the terms "strange" and "attractor". See, for instance, Devaney (1989, p. 211) and Medio (1992, pp. 46 and 158).

There is one Lyapunov exponent for each dimension in phase space. A positive Lyapunov exponent[12] measures how rapidly nearby points diverge from one another. On the contrary, a negative Lyapunov exponent measures how long it takes for a system to reestablish itself after a perturbation.

Deterministic chaos requires the largest Lyapunov exponent to be positive.

Let us suppose two initial conditions, x_0 and x'_0, as near one from the other as we want, and follow the trajectories $(F^k(x_0))$ and $F^k(x'_0)$, starting from x_0 and x'^0. Sensitive dependence on initial conditions means that nearby trajectories must diverge. The largest Lyapunov exponent measures the rate of local divergence and averages the rate over a typical long trajectory generated by the map F.

Suppose we make a small error in measuring the initial state and want to forecast the state one period from now. The largest Lyapunov exponent is a measure of how fast the initial measurement error multiplies into error in one's forecast.

For example, let us suppose the largest Lyapunov exponent was 0,05. This means we lose 0,05 bit of predictive power with each iteration. Therefore, if we could measure current conditions to 2 bits of accuracy, we would lose all predictive power after 40 iterations.

Inasmuch our measurements have a finite accuracy, errors of measurement are unavoidable. We can increase precision, adding more decimals to our measurement, thus reducing the value of the largest Lyapunov exponent and, then, the rhythm at which nearby trajectories diverge. But this only postpones the moment of the divergence. It would only disappear if we could get an infinite degree of precision, which means infinite information. That would be the cost for exact prediction in a system subject to chaotic behavior.

Limits to forecasting

Sensitive dependence on initial conditions means that the further out in time we go, the less accurate our forecast become. We do know the equations of motion, but the accuracy of

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[12] The Lyapunov exponent (L) is defined by:

$$L = \lim_{t \to \infty} \left[\ln \left(\| (F^t)'(x) . v \| \right) / t \right]$$

where $F^t(x)$ denotes the t-th iterate of F starting at initial condition x, i.e., $x_{t+1} = F^t(x)$, "'" is derivative, $\| \quad \|$ is a norm, v is a direction vector and "." denotes a scalar product.

the predictions depends on the quality of the inputs. Nonlinearity amplifies initial lack of precision until we become unable of predicting where a certain trajectory will be.

The inverse of Lyapunov exponent is called the Lyapunov time ($\tau = \frac{1}{L}$) and it measures after how many iterations the knowledge of initial conditions of the system is lost and its trajectory cannot be known.

In this respect, chaos implies the existence of a *temporal horizon* -defined by the Lyapunov time- after which our forecasts lose reliability at all.

Impredictibility is an intrinsic characteristic of chaotic systems. It cannot be eliminated by any finite increase in the accuracy of information. We can extend the Lyapunov time, i.e., the time during which a trajectory may be forecasted, increasing the precision of the measurement of initial conditions, but sooner or later we will be faced with diminishing marginal returns.

Randomness and determinism

One interesting question raised by this analysis is whether there exist truly random events. The question has early been raised in the survey article by Baumol and Benhabib (1989).

If a series generated by quite a deterministic equation looks perfectly random, is there anything like a truly stochastic process at all? If, "from the point of view of practice, there is no difference between high-dimensional deterministic chaos and randomness,"[13] which is the space left for the concept of a purely random process?[14]

For the time being, what can effectively be said is that there does not exist the sort of sharp opposition it was used to be thought to exist between determinism and randomness.

On the contrary, we are tempted to think of the existence of a continuum, where randomness appears as the extreme case of determinism or, if we prefer, deterministic chaos may be thought of as the *bridge* between simple determinism and pure stochasticity.

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[13] Brock, Hsieh, and LeBaron (1993, p. 14).

[14] A significant effort is being devoted to devise methodological procedures which may allow to distinguish a random series from one generated by deterministic chaos. The so called BDS statistic -named after its authors: Brock, Dechert, and Scheinkman- is an example of the advance in this direction. See Brock *et al*, *ibid*.

Up to now, Mathematics was the realm of determinism and Statistics, the kingdom of randomness. Now, we are inclined to think that nonlinear analysis and, particularly, chaotic dynamics represent an intermediate zone between one and the other field of knowledge. Perhaps, further research may show there are more things in common between them than what we presently think of.

Perhaps, in a future we may be forced to conclude that the relationship between determinism and randomness is one that resembles that existing in quantum theory between particles and waves.

As we have said, it has already been established that although we may improve the accuracy of the measurement of initial conditions, chaotic dynamics lead us, sooner or later, to face a temporal barrier beyond which no exact prediction is possible. Further on the Lyapunov time probabilities replace determinism. We can only predict, with a certain level of probability, that a certain trajectory will fall within a certain region but we are unable to forecast it with certainty as if it were a truly random trajectory. Beyond the temporal horizon, Statistics replaces Mathematics.

Conclusions

By means of a very simple model of demand and supply we have shown the consequences of the introduction of the assumption of a nonlinear behavior.

Basically, we are faced with a *more general* model than the linear one, inasmuch as it allows for different solutions which range from equilibrium up to chaos, depending on the values taken by the control parameters.

The main characteristic of nonlinearity is precisely that the *same* model may lead to qualitatively different results in response to qualitative parametric changes. In Hegelian terms, quantitative change becomes qualitative change.

In less philosophical terms, Peters (1991) illustrates the point as the straw that breaks the camel's back. In fact, as we add weight to the burden a camel is to carry, a point is reached where the animal cannot handle any more weight. A straw placed on the camel's back will cause the camel to collapse. The weight reaches a critical level at which the animal collapses.

In other words, the message is: in Economics *quantity matters*.

Postulating nonlinearity implies the belief that the portion of reality under analysis is better modeled by assuming a non-uniform response to changes in the independent(s) variable(s).

Linearity is a sort of pre-Columbian way of reasoning: it tells us that if we point to the West we can never reach the East. The size and direction of the response to equal changes in the exogenous variable(s) are always the same.

Of course, this sounds quite plausible for local analysis but it is a very particular and unusual case when the analysis is referred to big changes. The latter is the kingdom of nonlinearity.

Although we have restricted our analysis to just an exercise using the logistic equation, most of the results are valid for a wider field of applications.

First, one should recall that mappings that are topologically conjugate are completely equivalent in terms of their dynamics.

This means that, in particular, provided a function is single-peaked, has a negative Schwartzian derivative and is increasing in γ, then our results will go through.

The second reason is the following. Although a detailed mathematical theory has been developed so far only for one-dimensional dynamical systems, higher-dimension systems have been studied in particular cases or by means of computer simulations. These systems display the kinds of behavior discussed in this paper as well as other forms of complex behavior.

There is another argument in favor of nonlinear dynamic analysis.

If a visitor of Mars arrives at the Earth and in order to understand the world economy begins studying economic theory she would expect to find economic series randomly fluctuating around equilibrium or converging to steady states. She will be astonished when, analyzing the behavior of empirical variables, she will realize that "there is little if any evidence that economic date converge to stationary states, to steady growth or to periodic cycles."[15] This "corollary fact of *monumental importance* for the construction of economic science"[16] as Day calls it, emphasizes the importance of nonlinear models as a tool for studying economic change.

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[15] R.H. Day (1993, p. 3).
[16] Ibid.(the emphasis is ours).

Change is incompatible with equilibrium. If a system is in equilibrium it has no history; it is always in that state, but for randomly distributed shocks.

On the contrary, evolution is associated with *structural instability*.

Structural instability refers to perturbations in the function space. A dynamical system is said to be structurally stable if it is dynamically equivalent to a system sufficiently close to it in some sense.

On the contrary, a system is structurally unstable if a small perturbation is capable of yielding a qualitatively new dynamical behavior.

In particular, the qualitative change that maps undergo as parameters change is called *bifurcation*.

Thus, bifurcation theory studies structurally unstable dynamical systems. For instance, one of the major ways a map can be structurally unstable occurs when there is a lack of hyperbolicity[17]. Bifurcations occur, precisely, near non-hyperbolic fixed and periodic points.

Here it comes the relationship with the concept of evolution. Evolution is what we call the transit -from one mode of functioning to another- a dynamical system undergoes at a bifurcation point due to a parametric change.

Nonlinearity sheds a new light on the boundaries of comparative static analysis which, already pointed out by Samuelson's principle of correspondence, are not always well remembered by the members of the profession.

Static comparative analyses are legitimate provided equilibrium is stable and only within the limits of validity of that stability.

If equilibrium is locally stable, local will also be the scope of comparative statics analysis.

Nonlinearity implies that a change in a parameter value can lead the system to a new equilibrium point -in which case comparative static analysis holds- but it may also reach a point of bifurcation, a limit cycle, a chaotic map, etc.

A thorough review of nonlinear models applied to Economics may be found in the survey article by Boldrin and Woodford (1990) and in the excellent books by H. W. Lorenz

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[17] A periodic point p is hyperbolic if $\left|(F^n)'(p)\right| \neq 1$.

(1989), J. Barkley Rosser Jr. (1991) and Medio (1992). Some of their policy implications are analized in Bullard and Butler (1993). All these works show that, since the pioneering works by Goodwin in the thirties and after the second wave of nonlinear dynamic analysis led by Benhabib, Day and Grandmont in the early eighties, a significant development has taken place in this field during the last years.

It is still an open question the exact relationship existing between chance and determinism. Deterministic chaos appears as a *bridge* between pure stochasticity and pure determinism. This is only one of the various areas open to research in this promising field of nonlinear dynamics.

What is out of question is that nonlinearity provides a powerful tool to develop a thorough analysis of far-from-equilibrium economic systems and of the laws of motion that govern their evolution.

Undoubtedly, the next years will witness a nonlinear growth of the interest in the field we have already proposed to christen as "economic nonequilibrium" (see Beker, 1994).

APPENDIX

Let us analyze the relationship existing between the results obtained above for the nonlinear demand case and the one in the linear case as far as stability is concerned.

Let us suppose, for the linear case, that the demand and supply functions are the following:

$$D_t = a\, P_t + b$$

$$S_t = A\, P_t + B$$

and that the price adjustment equation is

$$P_{t+1} = k\,(D_t - S_t) = k\, E_t$$

as in the main text.

In this case, stability holds if and only if

$$-1 < 1 + k < 1$$

So, *instability* arises in the linear case in the following situations (assuming $k>0$ and $A>0$):

1) If $a<0$ and $k(a - A) \leq -2$.

2) If $a \geq 0$ and

2.1) $a \geq A$ or

2.2) $k(a - A) \leq -2$

Let us now compare these results with the ones obtained in the main text for the nonlinear demand function.

For this purpose, a will be the slope of the tangent to the demand curve at the point we choose to consider.

In our supply curve,the slope was represented by β, so β=A.

In the nonlinear example we have analized above, case 2.1) is excluded because the supply curve slope always exceeds the demand curve one whenever the latest is positive and equilibrium exists at a positive price.

Case 2.2) is also ruled out because of the assumption we have made that $k = \frac{1}{\beta}$.

Then, $k(a - A) = \frac{a}{A} - 1 > -2$.

So, the only one case of instability which may hold in our example is 1). It will happen whenever $|a| \geq |A|$, that is if the demand curve slope exceeds in absolute value the supply curve one. This coincides with the conclusion we have already arrived at for the nonlinear case: instability appears on the negative sloped branch of the demand curve when $\frac{\alpha}{\beta} \geq 3$. This condition is analytically equivalent to the former one.

In fact, for any equilibrium point P_t^*, on the negative sloping branch of the demand curve it holds that $a = \alpha\,(1 - 2P_t^*)$,

being

$$\alpha P_t^* (1 - P_t^*) - \beta P_t^* = 0$$

thus

$$P_t^* = 1 - \frac{\beta}{\alpha}$$

then

$$a = 2 - \frac{\alpha}{\beta}$$

If

$$\frac{\alpha}{\beta} \geq 3 \Rightarrow \left|\frac{a}{\beta}\right| = \left|\frac{a}{A}\right| \geq 1$$

q.e.d.

This result is just a particular case of the general rule stated by the Hartman-Grobman theorem which says that -under certain conditions- the *local* behavior of a nonlinear system is qualitatively similar to that of the linearized one (see Medio, 1992, p. 50).

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